

EXERCISES

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Show that f is linear if and only if $f(x+y) = f(x) + f(y)$ and $f(ax) = af(x)$ for all $x, y \in \mathbb{R}$ and $a \in \mathbb{R}$.

2. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Show that f is linear if and only if $f(x+y) = f(x) + f(y)$ and $f(1) = 1$.

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Show that f is linear if and only if $f(x+y) = f(x) + f(y)$ and $f(1) = 1$ and $f(0) = 0$.

4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Show that f is linear if and only if $f(x+y) = f(x) + f(y)$ and $f(1) = 1$ and $f(0) = 0$ and $f(2) = 2$.

5. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Show that f is linear if and only if $f(x+y) = f(x) + f(y)$ and $f(1) = 1$ and $f(0) = 0$ and $f(2) = 2$ and $f(3) = 3$.

	Linear	Not Linear
$f(x+y) = f(x) + f(y)$	Yes	No
$f(ax) = af(x)$	Yes	No
$f(1) = 1$	Yes	No
$f(0) = 0$	Yes	No
$f(2) = 2$	Yes	No
$f(3) = 3$	Yes	No

EXERCISES



Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a function. Show that f is linear if and only if $f(x+y) = f(x) + f(y)$ and $f(ax) = af(x)$ for all $x, y \in \mathbb{R}^3$ and $a \in \mathbb{R}$.