Expressions and Equations

In **Expressions and Equations**, students continue to develop their understanding of the order of operations with rational numbers. The **order of operations** is a set of rules for determining the order in which the operations in an expression are performed. A **rational number** is a number that can be expressed as a fraction, or ratio of two integers, where the denominator is not zero. Fractions, decimals, integers, and percents are all rational numbers. Students also continue to develop understanding of dependent and independent variables (letters that represent numbers), and use them to represent quantities in real-world or mathematical problems.

Additionally, students use the properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. **Linear expressions** are expressions in which no variable is raised to a power other than 1. For example, 3x + 6 is a linear expression. A coefficient is a number in front of a variable that indicates the quantity of the variable. For example, 3 is a coefficient in the expression 3x + 6, and 2 and 5 are coefficients in the equation 2x + 5y + 3 = 29.

At this level, students expand their mathematical versatility by using equivalent expressions to solve problems. Students learn that changing the form of an expression can shed light on its role in a problem and on how the quantities in the problem are related. Students assess the reasonableness of answers by using mental computation and estimation strategies.

#### The Grade 7 Common Core State Standards for Expressions and Equations specify that students should–

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

The following hands-on activities enable teachers to help students understand the principles of expressions and equations. The activities will help students use expressions and equations to flexibly tackle algebraic thinking and reasoning tasks.

## **Expressions and Equations**

## Contents

Lesson 1	Mixed Numbers, Decimals, and Percents Greater than 100%	60
	<b>re:</b> Write a number as a mixed number, a decimal, and a nt greater than 100%.	
Manipu	lative: Deluxe Rainbow Fraction <sup>®</sup> Squares	
Objectiv	Converting Fractions, Decimals, and Percentages re: Convert fractions into decimals and percentages. lative: Color Tiles	64
	Fraction, Decimal, and Percentage Combinations that Equal 1	68
<b>Objective:</b> Convert a percentage to a fraction.		
Manipu Circle	lative: Deluxe Rainbow Fraction <sup>®</sup> Circles; Rainbow Fractior Rings	٦
<b>Objectiv</b> proble	Solving Linear Equations ve: Write an appropriate linear equation for a given word em and find the solution. lative: Algeblocks®	72
	Problem Solving: Two-Step Linear Equations re: Write a two-step equation to solve a real-world proble	

Manipulative: Algeblocks®



Write a number as a mixed number, a decimal, and a percent greater than 100%.

## Common Core State Standards

**7.EE.3** Solve multi-step reallife and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

## **Expressions and Equations**

## Mixed Numbers, Decimals, and Percents Greater than 100%

Students expand their experiences with different number representations by looking at mixed numbers and their equivalent decimals and percents. Using models helps students increase their flexibility with these numbers. As their number fluency increases, students begin to differentiate between situations in which one representation may be more suitable than another.

Try It! Perform the Try It! activity on the next page.

## Talk About It

Discuss the Try It! activity.

- Ask: Could you compare the numbers in the same way if each person had used a different-size carton to hold the balls? Why or why not?
- Have students explain how to write  $1\frac{1}{12}$  as a percent.
- Have students describe situations in which it might be better to choose one form over another to represent a number. For example, when using money, a decimal is the more accepted number representation.

## Solve It

Reread the problem with students. Have students compare the fractional portions of their models to answer the question.

## More Ideas

For other ways to teach equivalency among mixed numbers, decimals, and percents greater than 100%—

- Have students use Fraction Tower<sup>®</sup> Equivalency Cubes to model the problem. Have students see that one whole equals 1, 1.0, and 100%, regardless of the number of equal-size parts in the whole. Have students show amounts greater than one whole as they explore how different values can be combined to show mixed numbers, decimals greater than one, and percents greater than 100%.
- Have students cover Base Ten Flats with Deluxe Rainbow Fraction<sup>®</sup> Squares to model amounts greater than one whole, then write each amount in the three forms using their knowledge of percents, decimals, and fractions. Some students may find it helpful to shade 10 × 10 grids (BLM 13) to show each amount and to help them to rewrite the amounts in each of the three forms.

## Formative Assessment

Have students try the following problem.

Which equation is true?

#### Try It! 20 minutes | Groups of 4

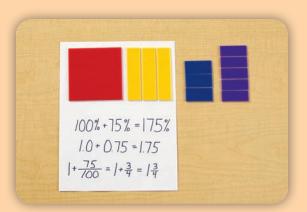
Here is a problem about mixed numbers, decimals, and percents.

Randi, Nick, and LaKeisha collected golf balls from a pond on the golf course and put them into cartons. Each carton holds the same number of balls. Randi filled  $1\frac{3}{8}$  cartons, Nick filled 1.5 cartons, and LaKeisha filled 175% of a carton. Who found the most golf balls?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials. Review the ways that one whole can be represented: 1, 1.0, and 100%. In the problem, a whole is one carton of golf balls.

# $I + \frac{3}{8} = I\frac{3}{8}$ I.0 + 0.375 = I.375I00% + 37.5% = I37.5%

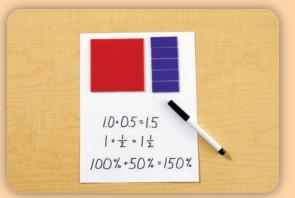
**1.** Write  $1 + \frac{3}{8} = 1\frac{3}{8}$  on the board. **Say:** A mixed number is the sum of its whole-number part and its fraction part. Use fraction pieces to show  $1\frac{3}{8}$ . Have students rename the fraction part as a decimal and as a percent, use the correct names for the whole, and write the corresponding equations.



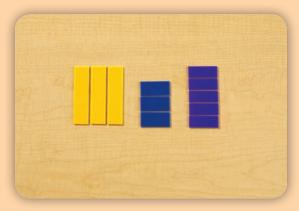
**3.** Write 100% + ? = 175% on the board. **Ask:** What percent can replace the question mark? Discuss how to use fraction pieces to show 75%. Have students model the equation and write three equations shown by the model.

#### Materials

- Deluxe Rainbow Fraction<sup>®</sup> Squares (1 set per group)
- paper (3 sheets per group)



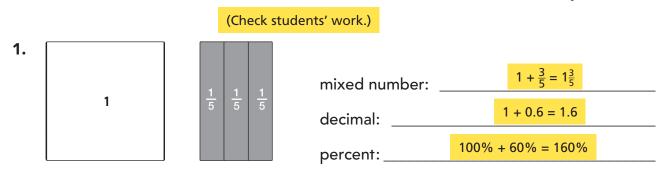
2. Have students show 1.5. Ask: How can we write an equation to add the whole-number part and the decimal part of this number? Write 1.0 + 0.5 = 1.5 on the board. Ask: How can we write five-tenths as a fraction in simplest form and as a percent? Have students write the three equations shown by the model.



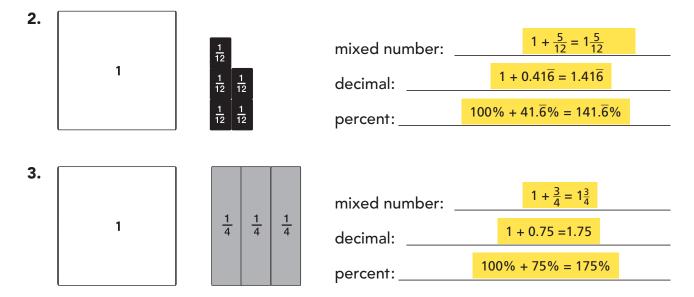
**4.** Have students recognize that the red square represents 1 and is the same in each case. Students can therefore answer the question by comparing the fractional portions of their models.



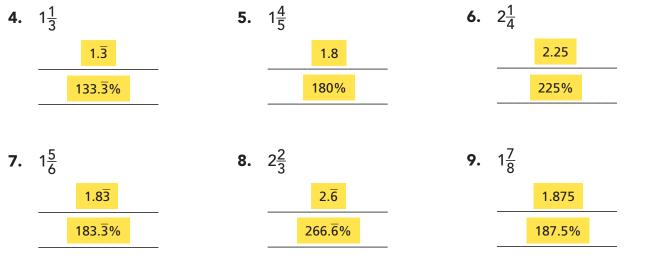
Use Fraction Squares to model each mixed number. Write a number sentence for the mixed number model. Write number sentences for the decimal and for the percent.



Using Fraction Squares, model each number. Write number sentences for the mixed number, decimal, and percent.



Write each mixed number as a decimal and as a percent.



**Challenge!** Compare the mixed numbers in Questions 1, 2, and 3. Write the numbers as decimals from greatest to least. Explain how you compared the numbers.

Challenge: (Sample) 1.75, 1.6,  $1.41\overline{6}$ ; I looked at the whole number. Because all were 1, I looked at the tenths place. 7 > 6 > 4





Convert fractions into decimals and percentages.

#### Common Core State Standards

**7.EE.3** Solve multi-step reallife and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

## **Expressions and Equations**

## Converting Fractions, Decimals, and Percentages

In previous grades, students learned that the set of rational numbers consists of all numbers of the form  $\frac{p}{q}$ , where p and q are integers and  $q \neq 0$ . Students should also be familiar with reducing fractions. In this lesson, students will use their previous knowledge of fractions to convert fractions to both decimals and percentages.

Try It! Perform the Try It! activity on the next page.

## **Talk About It**

Discuss the Try It! activity.

- Ask: What is a percentage? Elicit from students that it is the ratio of some number to 100. Ask: How do you change a fraction to a percentage?
- Ask: To change a decimal to a percentage, how many places should you move the decimal point? Then have students explain how to change a percentage to a decimal.

## Solve It

Reread the problem with students. Have students generate three reduced fractions from the story problem. Have them convert each fraction to a decimal and then each decimal to a percentage.

## More Ideas

For other ways to teach about fractions, decimals, and percentages-

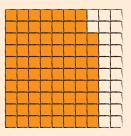
- Give students about 200 Centimeter Cubes in five colors. Have students randomly select 100 cubes, note the number of cubes of each color, express each color as a fraction of 100, reduce all fractions that are not in lowest terms, and convert each fraction to a decimal and to a percentage.
- Have students use Fraction Tower<sup>®</sup> Equivalency Cubes to solve similar problems. Any combination of cubes that can be stacked to the same height as the red cube will be equal to 100% (e.g.,  $\frac{1}{8}$ ,  $\frac{1}{8}$ ,  $\frac{1}{4}$ ,  $\frac{1}{6}$ , and  $\frac{1}{3}$ ). The other sides of the cubes show decimal and percent equivalents.

## **Formative Assessment**

Have students try the following problem.

Which of the following sets of fractions, decimals, and percentages is represented by the shaded area of the 10 × 10 grid shown here?

A.  $\frac{88}{100}$ , 0.88, 88%B.  $\frac{78}{100}$ , 0.39, 78%C.  $\frac{39}{50}$ , 0.78, 78%D.  $\frac{22}{25}$ , 0.88, 88%



#### Try It! 20 minutes | Groups of 4

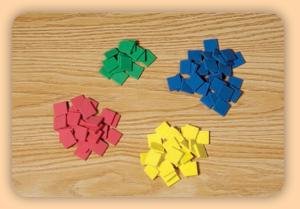
Here is a problem about fractions, decimals, and percentages.

A marketing company conducted a survey of one hundred 13- to 18-yearolds asking them to name their favorite type of movie. The results were as follows:

Action: 26 Science Fiction: 30

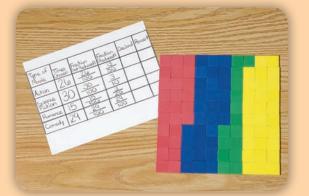
Romance: 15 Comedy: 29

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.



**1.** Have students form single-color groups of color tiles to reflect the number of responses to each survey category. Use the following color key:

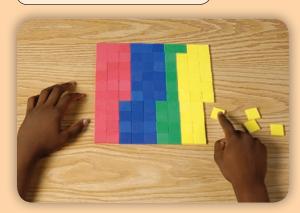
Action = red; Science Fiction = blue; Romance = green; Comedy = yellow.



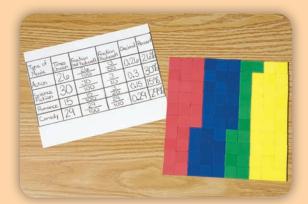
**3.** Have students make a table with six columns: Type of Movie, Times Chosen, Fraction (Not Reduced), Fraction (Reduced), Decimal, and Percent. Tell students to fill in the first two columns based on information from the problem and to fill in the fractions in the Not Reduced column by writing the number of times that type of film was chosen over 100. Finally, have students reduce the fractions.

## Materials

Color Tiles (33 of each color)



**2.** Have students form a 10-by-10 array with the tiles, grouping the colors to reflect the survey results.

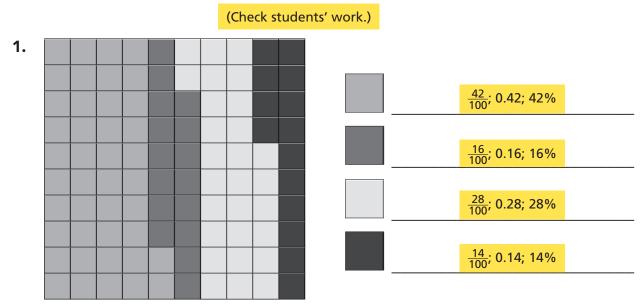


**4.** Have students write the decimal form for each result by looking at the numerator of the fractions that have not been reduced. Since 100 is the denominator, the decimal point should be put in front of any two-digit number (e.g.,  $\frac{22}{100}$  would be written as 0.22). Students can convert the decimal to a percentage by moving the decimal point two places to the right and writing a percent sign after the numeral.

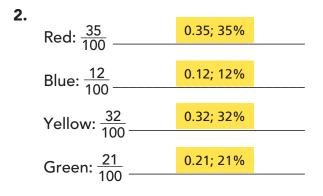




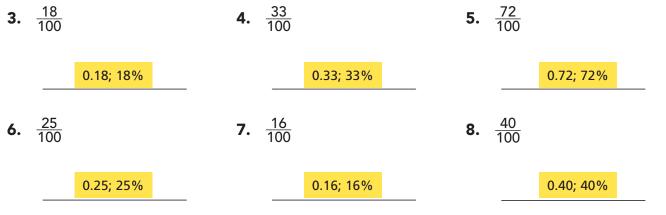
Use Color Tiles in a 10  $\times$  10 array to model the fraction shown. Write the fraction for each color. Then write the decimal and percent for each color.



Using Color Tiles, model a 10  $\times$  10 array for the fractions given. Sketch the model. Write the decimal and percent for each color.



#### Write each fraction as a decimal and as a percent.



**Challenge!** Why can you use a 10 × 10 array to convert a part of a total to a percent?

Challenge: (Sample) A  $10 \times 10$  array has 100 units. Percent means per one hundred.





Convert a percentage to a fraction.

### Common Core State Standards

7.EE.3 Solve multi-step reallife and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

## Expressions and Equations Fraction, Decimal, and Percentage Combinations that Equal 1

Previously, students learned to convert fractions to percentages. They divided the numerator by the denominator, multiplied the quotient by 100, and added a percent sign. In this lesson, students will work these steps backward to convert percentages to fractions.

Try It! Perform the Try It! activity on the next page.

## Talk About It

Discuss the Try It! activity.

- Ask: What fraction of the circle does each section represent?
- Ask: What is the decimal equivalent of each of the fractions?
- Have students explain how to convert a percentage to a decimal and then a decimal to a fraction without using the Rainbow Fraction Circle Rings.

## Solve It

Reread the problem with students. Have them convert the percentages to decimals. Then have students use the Rainbow Fraction<sup>®</sup> Circle Rings to determine the equivalent fractions in order to solve the story problem.

## **More Ideas**

For another way to teach about fractions, decimals, and percentages-

Students can repeat this activity using Fraction Tower<sup>®</sup> Equivalency Cubes to represent the various types of glass found in Roberto's artwork.

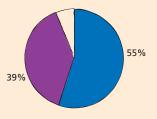
## **Formative Assessment**

Have students try the following problem.

Which of the following sets of a fraction, decimal, and percentage represents the missing section of the circle graph shown here?

**A.**  $\frac{6}{100}$ , .6, 6% **B.**  $\frac{3}{50}$ , .06, 6%

**C.**  $\frac{8}{50}$ , .08, 16% **D.**  $\frac{16}{100}$ , .16, 16%

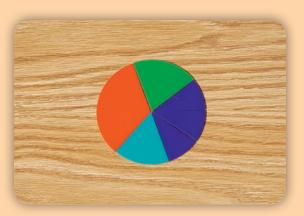




Here is a problem about fractions, decimals, and percentages.

Roberto is designing a circular glass work of art for his Fine Arts class. He would like the circle to be divided into four sections, each with a different type of glass. The sections should be 20% beveled glass, 16.6% swirled glass, 30% bubbled glass, and  $33.\overline{3}$ % wrinkled glass. What fraction of the circle will each texture be?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.



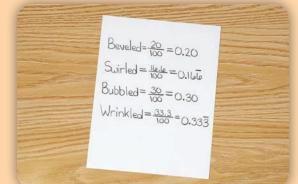
**1.** Have students use the Fraction Circles to represent the percentages for each type of glass. The Percent Ring will help students choose the appropriate pieces. **Ask:** What Fraction Circle piece will you use to represent the beveled glass section? The wrinkled glass section? How will you represent the bubbled glass?



**3.** Have students use the Fraction Circle Rings to determine which fraction each decimal represents. Have students record their results on a sheet of paper.

#### Materials

- Deluxe Rainbow Fraction<sup>®</sup> Circles
- Rainbow Fraction Circle Rings

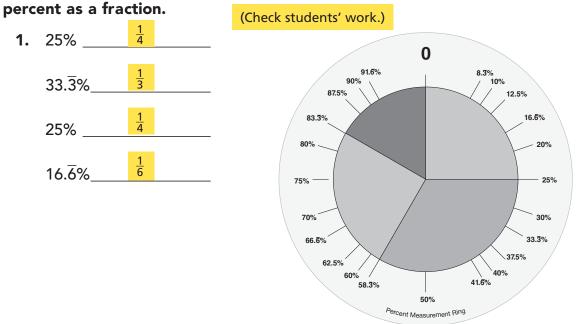


2. Now have students convert the percentages to decimals. Ask: Which way should you move the decimal point, left or right? How many places should you move it?

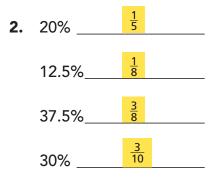
## Look Out!

Some students may add the  $16.\overline{6}\%$  and the  $33.\overline{3}\%$  and write the sum as  $49.\overline{9}\%$ . Explain to them that the repeating bar (<sup>-</sup>) indicates that the last digit repeats endlessly and that the number is therefore an approximation. These percentages represent  $\frac{1}{6}$  and  $\frac{1}{3}$ , which add up to  $\frac{1}{2}$ . It is understood that in these situations, the . $\overline{6}$  and the . $\overline{3}$  equal 1 when added, and so  $16.\overline{6}\%$  plus  $33.\overline{3}\%$  equals 50.0%.

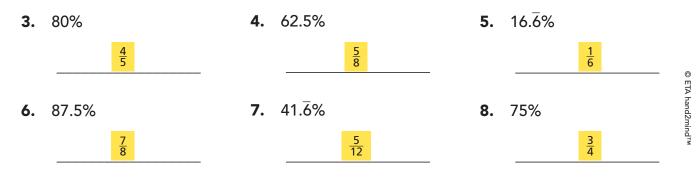
## Use Fraction Circles and Fraction Circle Rings to model each percentage. Write the



#### Using Fraction Circles and Fraction Circle Rings, model each percentage. Sketch the model. Write the percent as a fraction.



#### Write each percent as a fraction.



**Challenge!** What does the word *percent* mean? Explain how to get the numerator and decimal of a fraction equivalent to a given percent.

Challenge: (Sample) *Percent* means per 100. The denominator of the fraction equivalent to a percent is 100. The numerator is the number of the percent written without a percent symbol.





Write an appropriate linear equation for a given word problem and find the solution.

### Common Core State Standards

**7.EE.4a** Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## **Expressions and Equations**

## **Solving Linear Equations**

Students have learned that a linear equation is made up of up to 2 polynomials or monomials that have variables of only the first degree. They know how to determine whether a given equation is linear and how to solve linear equations with one variable. This lesson focuses on how to solve applications of linear equations with variables on both sides of the equal sign.

Try It! Perform the Try It! activity on the next page.

## **Talk About It**

Discuss the Try It! activity.

- Ask: Is the amount of money the children have expressed in terms of Corina or in terms of Matsu?
- Ask: Are there any like terms you can combine on either side?
- Ask: If you take (subtract) an x-block from the left side and an x-block from the right side, will the balance hold? What if you take a second x-block from each side?

### Solve It

Reread the problem with students. **Ask:** What should you do now that you know that the x-block is worth \$3? Help students understand that "x = 3" means that Matsu has \$3. Since Corina has 3 more than 4 times that much, she has \$15.

## **More Ideas**

For other ways to teach about equations—

- Have students solve the problem using Algebra Tiles<sup>™</sup>.
- Extend the activity by having students solve 3x 8 = 2 2x and 6x + 5 = 8x 1.

## **Formative Assessment**

Have students try the following problem.

Solve the given equation.

3x + 7 = 5x + 3

**A.** 2x + 3 = 7 **B.** x = 2 **C.** 3x + 4 = 5x **D.** x = 4

#### Try It! 15 minutes | Pairs

Here is a problem about writing equations.

Corina has \$3 more than four times the amount of money that Matsu has. Together they have six times the amount that Matsu has. How much money does each child have?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.



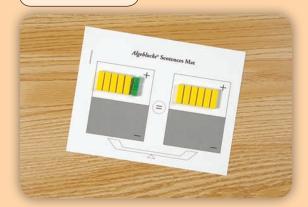
1. Help students model the amount of money from the information given in the word problem. **Ask:** How can you express the amount of money that Matsu has? How can you express the amount of money that Corina has? How can you express the amount of money that they have together?



**3.** Ask students to take a yellow *x*-block from the left side at the same time that they take an *x*-block from the right side until their mat shows 3 green unit blocks on the left and 1 *x*-block on the right side. **Ask:** What is the value of x? Have students find the solution and write the answer on a piece of paper.

Materials

Algeblocks<sup>®</sup>
BLM 7



**2.** Have students write the equation on a piece of paper. Have students place Algeblocks on the Sentences Mat to create a model of the linear equation and show that together they have six times as much as Matsu.

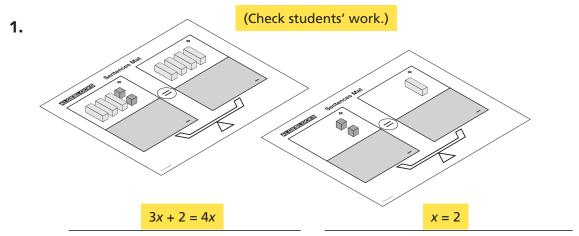
## A Look Out!

Some students will stop working when they have found the value of x. Ask them to reread the problem to see what question is being asked. **Ask:** *Does each child have \$3?* Explain to students that they must take the x-value and apply it to the word problem in order to determine the amount of money that each child has.



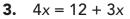


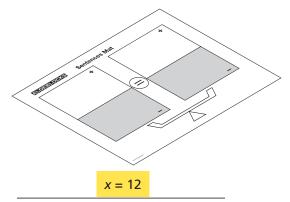
## Use Algeblocks and an Algeblocks Sentences Mat to model the equation shown and then solve it. Write the equation and the solution.



Using Algeblocks and an Algeblocks Sentences Mat, model each equation. Sketch the model. Write each solution.

2. 3x + 9 = 4x 3.





#### Find each solution.

**4.** x + 1 = 2x

*x* = 1

**6.** 6x + 5 = 7x

*x* = 5

8. 10x = 9x + 9

**5.** 4x = 1 + 3x

- *x* = 1
- **7.** x + 6 = 2x

*x* = 6

**9.** 8x + 8 = 7x

© ETA hand2mind™

**Challenge!** When solving an equation, how do you get both the variable terms on the same side of the equal sign? Explain.

Challenge: (Sample) Add the opposite of the term that is with the constant term to both sides of the equation.





Write a two-step equation to solve a real-world problem.

#### Common Core State Standards

■ 7.EE.4a Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## **Expressions and Equations**

## Problem Solving: Two-Step Linear Equations

Up until now, students have been working with one-step linear equations. This activity demonstrates the use of Algeblocks<sup>®</sup> as an aid to representing and solving two-step linear equations.

Try it! Perform the Try It! activity on the next page.

## Talk About It

Discuss the Try It! activity.

- Ask: How do we know that 19 = 4b + 7 is the correct equation? What does the 19 represent? What does the 4b represent? What does the 7 represent?
- Ask: What can we remove from both sides of the equation that will still allow the equation to stay balanced?
- Ask: If 4 blocks equal 12, what does each block equal? What does the block represent?
- Ask: What does b represent? How many bushes did Lauren trim?

## Solve It

Reread the problem with students. **Say:** *Explain how you determined the number of bushes Lauren trimmed.* **Ask:** *How much would Lauren earn if she trimmed 6 bushes?* **12** *bushes?* 

## **More Ideas**

For another way to teach about two-step linear equations-

Have students create a table of values and use the XY Coordinate Pegboard to display a graph that shows how much Lauren can earn for mowing larger lawns.

## **Formative Assessment**

Have students try the following problem.

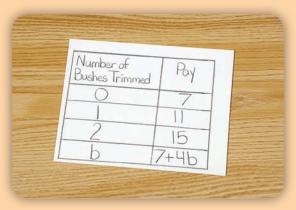
Solve for y. 3y - 12 = y - 18 A. y = 5 B. y = -5 C. y = -3 D. y = 3

### Try It! 15 minutes | Pairs

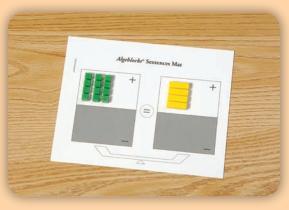
Here is a problem about solving a two-step equation.

During the summer, Lauren mows her neighbor's lawn. She is paid \$7 for mowing the lawn and an additional \$4 for each bush she trims. If she was paid \$19, how many bushes did she trim?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

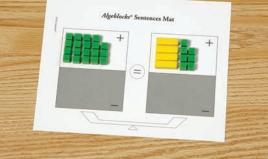


**1.** Have students create a table that reflects the information given in the story problem. Use this table to generate an equation.



**3.** Have students remove 7 unit blocks from each side by making zero pairs, leaving them with 12 unit cubes on the left and 4 *b* blocks on the right. These should be arranged in equal groups as shown. From here, have students divide both sides by the number of equal groups (4). The result is b = 3. Remind students that *b* represents the number of bushes that Lauren trimmed.





**2.** Have students represent this equation with Algeblocks, using the yellow *x* blocks to represent the number of bushes (*b*) Lauren trimmed and the green unit blocks for dollar amounts.

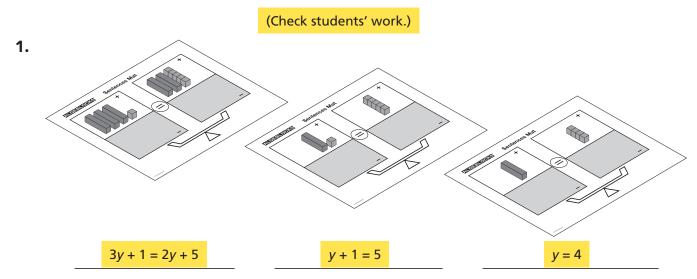


Watch for students who do not reverse the order of operations when solving the equation. Remind students that when solving equations they are "undoing" the operations.



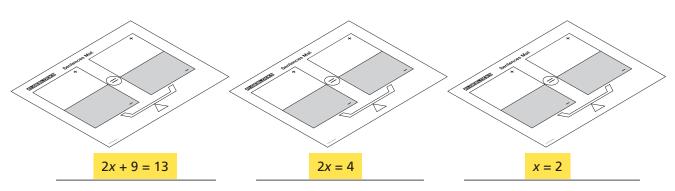
**Answer Key** 

Use Algeblocks and an Algeblocks Sentences Mat to model the equation shown. Write the equation. Write the equation after the first step and write the solution.



Using Algeblocks and an Algeblocks Sentences Mat, model the equation. Sketch the model, the first step, and the solution.

**2.** 2x + 9 = 13



Solve each equation.

**3.** 4x + 10 = 9x

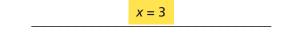
*x* = 2

y = 5

y = 2

- **5.** 6y + 10 = 8y
- **7.** 10y = 6y + 8





y = 1

- **6.** 2x + 12 = 5x
- **8.** 4*y* + 3 = 7*y*

© ETA hand2mind<sup>™</sup>

**Challenge!** Describe the two steps you used to solve the equations on the previous page. Choose an equation, show the step, and explain the reason for each step.

Challenge: (Sample) The first step is to add or subtract one of the constant terms from both sides. The second step is to divide both sides of the equation by the coefficient on the variable term.

