

Mathematical Induction (M.I.)

Prerequisites: Discrete Mathematics, Logic, Set Theory

Mathematical Induction: A Powerful Tool for Proving Statements

Principle:



Inductive Step:



Base Case:



Inductive Step:

Example 1: Proving the sum of the first n natural numbers.

Statement: $P(n) = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$

Base Case: $P(1) = 1 = \frac{1(1+1)}{2} = 1$ is true.

Inductive Step: Assume $P(k) = 1 + 2 + \dots + k = \frac{k(k+1)}{2}$ is true. We need to show $P(k+1) = 1 + 2 + \dots + (k+1) = \frac{(k+1)(k+2)}{2}$ is true.

Proof: $P(k+1) = 1 + 2 + \dots + k + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2} = P(k+1)$. \square

Applications of Mathematical Induction

Statement	Base Case	Inductive Step
$1 + 2 + \dots + n = \frac{n(n+1)}{2}$	$P(1) = 1 = \frac{1(1+1)}{2} = 1$	Assume $P(k) = \frac{k(k+1)}{2}$. Then $P(k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$.
$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$	$P(1) = 1 = \frac{1(1+1)(2(1)+1)}{6} = 1$	Assume $P(k) = \frac{k(k+1)(2k+1)}{6}$. Then $P(k+1) = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$.

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$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$	$P(1) = 1 = \frac{1(1+1)(2(1)+1)}{6} = 1$	Assume $P(k) = \frac{k(k+1)(2k+1)}{6}$. Then $P(k+1) = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$.