

Objective

Identify the constant of proportionality for a proportional relationship.

Common Core State Standards

- 7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Ratios and Proportional Relationships

Constant of Proportionality

A relationship is proportional if its graph is a straight line through the origin. The slope of this line is the constant of proportionality, or unit rate, for the relationship. The unit rate is also revealed on the line by the point (1, r), where ris the unit rate. Graphing a proportional relationship is the most powerful way to help students visualize what the constant of proportionality represents.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- Say: The problem says the number of blocks is proportional to the number of minutes, so the number of blocks goes on the y-axis and time goes on the x-axis.
- Explain that the calculation of the constant of proportionality will produce the same result regardless of the two points chosen from the line. Ask: Which point on the line will give you the constant of proportionality directly, without having to calculate? Elicit that the answer is the point whose x-coordinate is 1. That point is (1, r), where r is the unit rate, or constant of proportionality.

Solve It

Reread the problem with the students. Have students show the graph on grid paper. Ask them to find the vertical distance and the horizontal distance between the two given points and divide to find the constant of proportionality. Help them recognize that the constant is the *y*-value that the line crosses at x = 1.

More Ideas

For another way to teach about the constant of proportionality-

Have students use Fraction Tower[®] Equivalency Cubes to solve the problem. Start by asking students to build a whole tenths tower and a whole fifths tower. Have students lay down the fifths tower horizontally and then lay down the tenths tower horizontally immediately below it. Tell students the towers represent the ratio 5:10 (5 blocks in 10 minutes) because there are 5 fifths and 10 tenths. Next have students break the towers to show the ratio 2:4, then 1:2. Elicit from students that to find the unit rate, they need to find the number of green cubes for each purple cube. Help students see that they need $\frac{1}{2}$ of a green cube for each purple cube and that this $\frac{1}{2}$ is the unit rate, or constant of proportionality in the problem.

Formative Assessment

Have students try the following problem.

Teo has 3 trophies and 6 medals. Brandi has 5 trophies and 10 medals. If the number of medals is proportional to the number of trophies, which point will be on the graph of this relationship?

A. (0, 1) B. (1, 2) C. (4, 7) D. (12, 5)

Try It! 15 minutes | Pairs

Here is a problem about proportional relationships and the constant of proportionality.

Liam took his dog for a walk. In 4 minutes, he had walked 2 blocks. In 10 minutes, he had walked 5 blocks. If the number of blocks is proportional to the number of minutes, what is the constant of proportionality for the relationship?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.



1. Say: Let's graph the relationship on the pegboard. Put distance on the y-axis and time on the x-axis. **Ask:** What two points will you graph? Have students place pegs at (4, 2) and (10, 5) and connect the points with a rubber band. **Ask:** Can you confirm that the relationship is proportional? Ask students to extend their bands to the origin. Elicit that the relationship is proportional.



3. Say: Divide the vertical distance by the horizontal distance. The quotient is called the constant of proportionality. It is also called the unit rate, because it tells how much the y-coordinate changes for each unit of change in the x-coordinate. Have students confirm the unit rate by inspecting the graph.





2. Ask: What is the horizontal distance between the two points? Have students use a rubber band and peg to mark the horizontal distance between (4, 2) and (10, 5). **Ask:** What is the vertical distance between the two points? Have students use another band to mark the vertical distance between (4, 2) and (10, 5).

A Look Out!

Students might have difficulty finding the vertical and horizontal distances between points. Point out that the rubber bands that represent the distances form a right triangle with the line.





Use an XY Coordinate Pegboard. Build the model and use it to answer the question.

(Check students' work.)

1. For every 2 apples in Kali's orchard, there are 4 pears. In Sam's orchard, there are 10 pears for every 5 apples. If the number of pears is proportional to the number of apples, what is the constant of proportionality?





Using an XY Coordinate Pegboard, model the problem. Draw the model and use it to answer the question.

2. Yesterday, Maria had 4 nickels and 3 dimes in her wallet. Today, she has 12 nickels and 9 dimes. If the number of dimes is proportional to the number of nickels, what is the constant of proportionality?





Use Centimeter Grid Paper to find the constant of proportionality for the relationship.

3. Carson can jump 30 times in 18 seconds. He can jump 20 times in 12 seconds. Assume that the number of times Carson can jump is proportional to the number of seconds he is given.

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- 4. Cleo bought 36 stamps and paid \$12. Ned paid \$6 for 18 stamps. Assume the amount paid for stamps is proportional to the number of stamps purchased. $\frac{1}{3}$
- 5. Sheila made 14 threepoint shots in 35 attempts. She made 32 three-point shots in 80 attempts. Assume the number of three-point shots Sheila makes is proportional to her number of attempts.

 $\frac{2}{5}$

Answer Key

Challenge! Caitlyn read 2 books in 8 days. She read 4 books in 16 days. Assume that the number of books Caitlyn reads is proportional to the number of days she spends reading. Find the constant of proportionality for the relationship. Identify the point on a graph of the relationship that directly names the value of the constant. Explain why that point works.

Challenge: The two points represented in the problem are (8, 2) and (16, 4). The vertical distance between them is 4 - 2 = 2. The horizontal distance is 16 - 8 = 8. The constant of proportionality is $\frac{1}{4}$. The point that directly names the constant is $(1, \frac{1}{4})$. This is so because the constant of proportionality is the change in *y* for each unit change in *x*. The point $(1, \frac{1}{4})$ represents a 1-unit change in *x* relative to the point (0, 0). The vertical change is $\frac{1}{4} - 0 = \frac{1}{4}$ and the horizontal change is 1 - 0 = 1, so the *y*-coordinate of the point $(1, \frac{1}{4})$ literally names the constant of proportionality.

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Name

Centimeter Grid

BLM

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