Geometry

In eighth grade, students explore angles and incorporate ideas about similarity and congruency to describe and analyze two-dimensional figures. Two shapes are **similar** when their corresponding angles are equal and their sides are in proportion–they have the same shape, but not necessarily the same size. Sometimes shapes need to be rotated (turned), reflected (flipped), translated (slid), or dilated (resized) to actually determine whether or not they are similar.

Shapes are **congruent** when they are the same shape and size. Whether they are rotated, reflected, or translated, they maintain their size and shape. For example, an equilateral triangle with a side length of 3 inches is similar to an equilateral triangle with a side length of 5 inches. However, it is congruent only to another equilateral triangle with a side length of 3 inches.

Students will learn and apply the **Pythagorean Theorem**, which describes the relationship between the three sides of a right triangle. The theorem is stated mathematically for a triangle having side lengths *a*, *b*, and *c* as $a^2 + b^2 = c^2$, where *c* is the hypotenuse. The Pythagorean theorem can be applied in many practical ways (e.g., in construction). Knowing that a triangle has a right angle helps in calculating an unknown length of a side; conversely, knowing the lengths of the sides can help in determining whether a triangle has a right angle.

The Grade 8 Common Core State Standards for Geometry specify that students should–

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

The following hands-on activities will help students learn geometry concepts in a meaningful way. Mathematically proficient students explain correspondences between equations and representations. They ask whether an answer makes sense within the context of a problem they are solving. Teachers should facilitate this process. For example, teachers will want to help students understand the meaning behind the Pythagorean theorem, and not merely teach the application of the formula.

Geometry

Contents

LESSON 1

Objective

Identify congruent figures.

Common Core State Standards

- **8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Geometry

Congruent Figures and Transformations

Congruent figures are exactly the same size and shape. Manipulatives help students visualize this. They can match figures and then move the figures to different positions and orientations, noting that the model does not change. They also can reverse the process to determine if figures in different positions are congruent.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How can you tell if two figures are congruent?*
- **Ask:** *Are figures still congruent when they are moved in more than one way?* Have students show two transformations, such as a reflection and a rotation, to verify their responses.
- Have students identify congruent figures in the classroom and recognize how different positions could be the result of translations, reflections, rotations, or combinations of transformations.

Solve It

Reread the problem with students. Have them draw a design for Calvin's window using four congruent rectangles. Then have them explain how to tell that the figures are congruent.

More Ideas

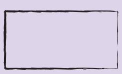
For other ways to teach about congruency and transformations—

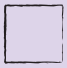
- Use Pattern Blocks to show congruency. Have students use different shapes to make congruent hexagons. Have them move the shapes to show that position and orientation do not affect congruency.
- Adapt the problem to use a different shape, such as a triangle. Have students use AngLegs® to solve the problem.

Formative Assessment

Have students try the following problem.

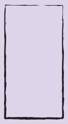
Which shape is congruent to this rectangle?



A. 

B. 

C. 

D. 

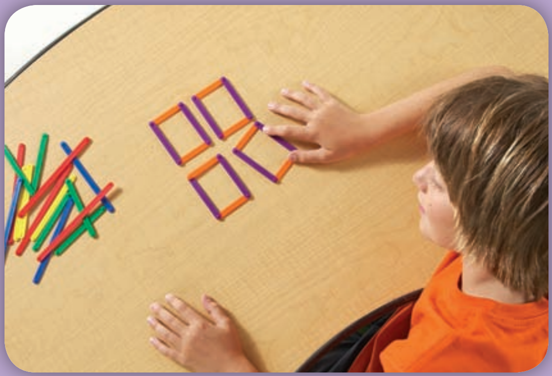
Try It! 25 minutes | Groups of 4

Here is a problem about congruent figures and transformations.

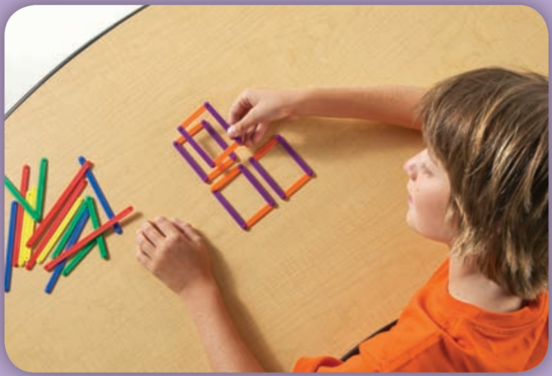
Calvin makes stained-glass windows. He lays out 4 rectangles to be used in one of his designs. The rectangles are arranged in two rows and two columns. How can he move the rectangles to determine if they are congruent?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials. **Say:** *Remember there are three types of transformations: translations, or slides; reflections, or flips; and rotations, or turns.* Draw the arrangement of the rectangles on the board.

Materials
- AngLegs® (1 set per group)



1. Have students use AngLegs to make a rectangle. **Say:** *Make three more rectangles that are the same size and shape as the one you just made.* Have students arrange the four rectangles as shown on the board.



2. Have students slide the top right-hand model to the left on top of the other rectangle. **Say:** *Compare the two rectangles. Are they the same size and shape?*

3. Have students reflect the bottom left rectangle onto the top rectangle. Then have them rotate the last rectangle onto the top of the stack of rectangles. Guide students to see that the size and shape of each rectangle remains the same when the position and/or orientation is changed. Explain that the figures are *congruent* because they have exactly the same size and shape.

Look Out!

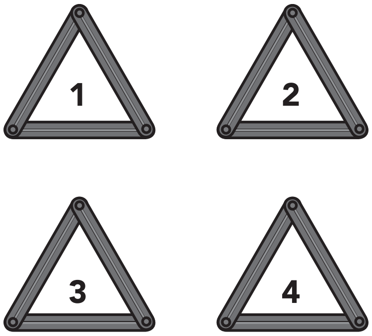
Some students may have difficulty maintaining the shape of the AngLegs when moving the models. You might need to have them brace the figure by connecting one or more diagonals. Remind them that the brace does not change the characteristics of the larger shape. Students who think that position and orientation affect congruency may need more hands-on practice with a variety of shapes.

Answer Key

Use AngLegs to model each set of shapes. Complete each statement with the phrase "is" or "is not."

(Check students' work.)

1.

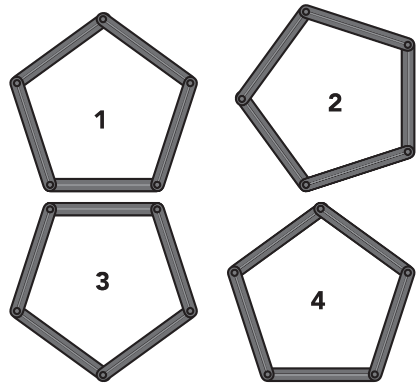


Triangle 1 ___is___ congruent to Triangle 2.

Triangle 2 ___is___ congruent to Triangle 3.

Triangle 3 ___is___ congruent to Triangle 4.

2.



Pentagon 1 ___is___ congruent to Pentagon 4.

Pentagon 3 ___is___ congruent to Pentagon 2.

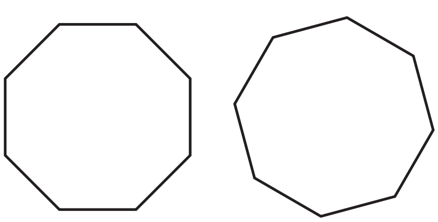
Pentagon 4 ___is___ congruent to Pentagon 3.

Using AngLegs, build three figures that are congruent to each other and one model that is not congruent to the others. Sketch the models. Write three statements about the figures' congruency.

3. Students' models will vary.

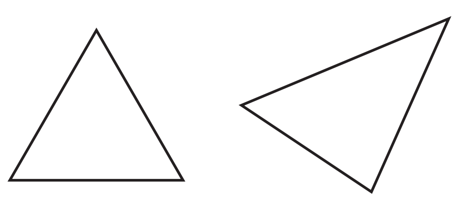
Tell if each pair of figures are congruent.

4.



___congruent___

5.



___not congruent___

Answer Key

Challenge! Why can you use the transformation of a slide to determine if your figures are congruent? Explain the process. Draw a picture to help.

Challenge: (Sample) A slide gets the figure oriented the same direction. You can slide the figure on top of the other to see if they are an identical match.

LESSON 2

Objective

Understand the terms *reflection* of a geometric figure and *line of reflection.*

Common Core State Standards

- **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Geometry

Reflections (Flips)

Students can expand their understanding of congruence by studying flips (reflections), turns (rotations), and slides (translations). A reflection "flips" a figure over a line called the *line of reflection*. A *reflection* over a line is a transformation in which each point of the original figure has a corresponding point that is the same distance from the line of reflection, but on the opposite side of the line.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Does the figure you formed by reflecting it look the same as the original figure? How is it like the original? How is it different?*
- **Ask:** *How do the coordinates change when the pre-image (the original image) is reflected over the* y*-axis?* (The *x*-coordinate on the image is the opposite of the *x*-coordinate of the pre-image.) *How would the coordinates change if the pre-image were reflected over the* x*-axis?*
- **Ask:** *How far are the points on the pre-image from the* y*-axis (line of reflection)? How far are the points on the image from the* y*-axis (line of reflection)?*

Solve It

Reread the problem with the students. Have them draw a diagram of the two flower gardens in Mr. Lanka's yard, indicating the line of reflection. Ask them to explain what makes this transformation a reflection (flip) over a line of reflection.

More Ideas

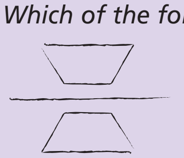
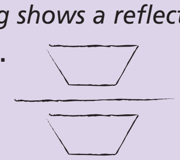
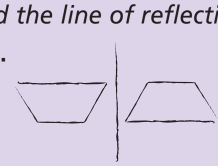
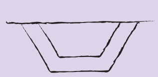
For another way to teach about reflections (flips)—

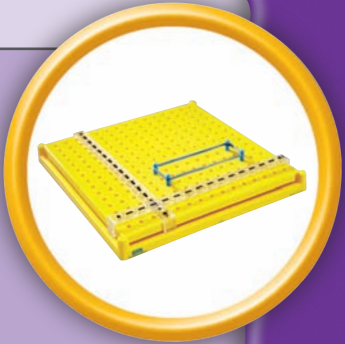
- Have students plot two geometric figures on the XY Coordinate Pegboard (Quadrants I and II as before) and tell whether or not the figures represent a reflection. If they do, have them indicate the line of reflection. The points are as follows: *A*(–7, 4), *B*(–6, 6), *C*(–4, 6), *D*(–3, 4); *A′*(3, 4), *B′*(4, 6), *C′*(6, 6), *D′*(7, 4).

Formative Assessment

Have students try the following problem.

Which of the following shows a reflection and the line of reflection?

A.  **B.**  **C.**  **D.** 



Try It!

30 minutes | Pairs

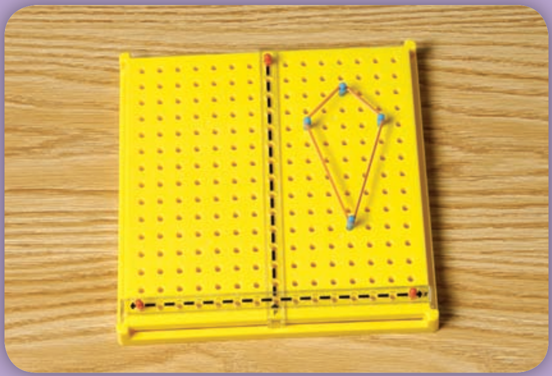
Here is a problem about reflections and lines of reflection.

Mr. Lanka has a large flower garden shaped like a kite on one side of his yard. He wants to be able to stand in the middle of the yard and look to one side and then the other and see the same shape, just as if he were looking in a mirror. What will Mr. Lanka's landscape plan look like with both flower gardens?

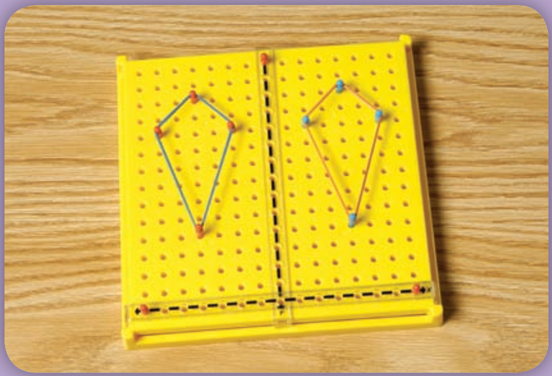
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

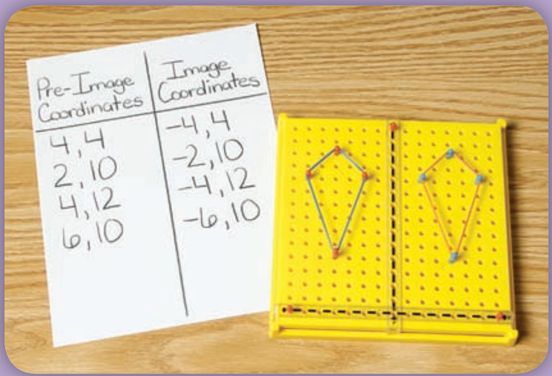
- XY Coordinate Pegboard
- paper (1 sheet per pair)
- pencils (1 per pair)



1. Have students set up the pegboard for graphing in Quadrants I and II. Have students use blue pegs to plot the following points: (4, 4), (2, 10), (4, 12), and (6, 10). Students should connect the points with a rubber band. These represent the location of Mr. Lanka's kite-shaped garden.



2. Have students reflect the image of the kite over the *y*-axis (the line of reflection) so that it is in the same position in Quadrant II as it is in Quadrant I. Have students use red pegs to plot this reflected image. Students should connect the points with a rubber band.



3. Have students compare the vertices of the pre-image to those of the image. Have students investigate the effect that reflecting a figure over the *y*-axis has on its coordinates.

⚠ Look Out!

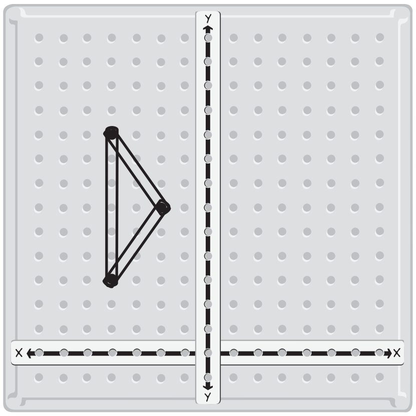
Watch for students who transpose coordinates and plot the *y*-coordinate first, rather than after the *x*-coordinate. Instruct students to start at the origin (0, 0). Remind them that the ordered pairs indicate how far to move horizontally and then vertically from the origin. Work through several examples that demonstrate the importance of order.

Answer Key

Use an XY Coordinate Pegboard to model each triangle. Write the coordinates of the vertices. Reflect over the axis shown. Name the coordinates of vertices for the new triangle.

(Check students' work.)

1.



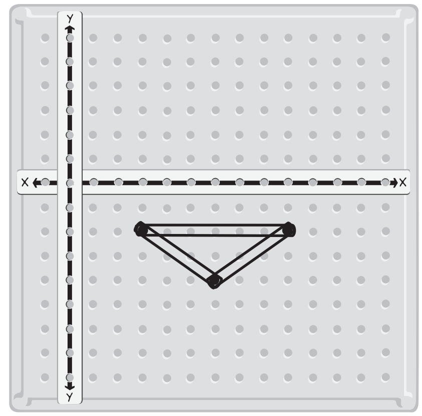
vertices of triangle

(–4, 3), (–2, 6), (–4, 9)

vertices of reflected triangle

y-axis; (4, 3), (2, 6), (4, 9)

2.



vertices of triangle

(3, –2), (9, –2), (6, –4)

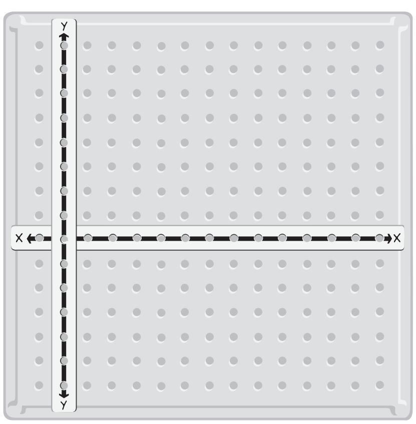
vertices of reflected triangle

x-axis; (3, 2), (9, 2), (6, 4)

Using an XY Coordinate Pegboard, build each polygon. Then model each reflection. Draw the reflection on the grid below. Name the coordinates of the new polygon. How do the coordinates change?

(Check students' models.)

3. rectangle with vertices at (2, 3), (2, 5), (4, 3), (4, 5) reflected over the *x*-axis

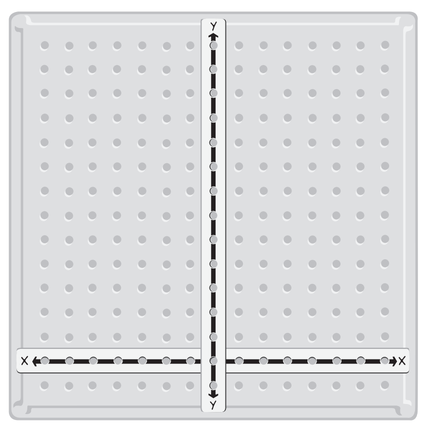


vertices of reflected triangle

(2, –3), (2, –5), (4, –3), (4, –5)

y coordinates are opposites

4. triangle with vertices at (3, 0), (4, 5), (1, 13) reflected over the *y*-axis



vertices of reflected triangle

(–3, 0), (–4, 5), (–1, 13)

x coordinates are opposites

Answer Key

Challenge! How do you use the line of reflection to find the location of the reflected figure?

Challenge: (Sample) From the line of reflection, the original figure and the reflected figure have to be the same number of units away, but on opposite sides of the line.

LESSON 3

Objective

Translate a geometric figure on a coordinate system.

Common Core State Standards

- **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Geometry

Translations (Slides): The Traveling Triangle

A *translation* is a transformation that moves each point of a figure the same distance and in the same direction. The figure that is created is congruent to the original figure and has the same orientation. This lesson allows students to visualize the "movement" as a figure is translated several times.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Does the figure you formed by translating (sliding) the figure look the same as the original figure? How is it like the original (pre-image)? How is it different?*
- **Ask:** *What does it mean for a figure to maintain its orientation as it slides?*
- **Ask:** *How will sliding a figure down 1 unit and right 2 units change the coordinates of the vertices?* Elicit that the *x*-value will increase by 2 while the *y*-value decreases by 1.

Solve It

Reread the problem with students. Ask them to explain how the figure in the story problem was translated from one position to the next.

More Ideas

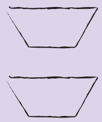
For other ways to teach about translations—

- Have students assemble a figure from AngLegs® pieces and use it to demonstrate a translation by tracing it, moving it in a prescribed way, tracing it again, and so forth.
- Have students place a Centimeter Cube at (0, 0) on grid paper and translate the cube to (8, 12) by moving it in steps of "up 3 and right 2."

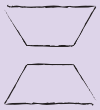
Formative Assessment

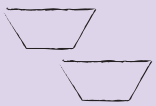
Have students try the following problem.

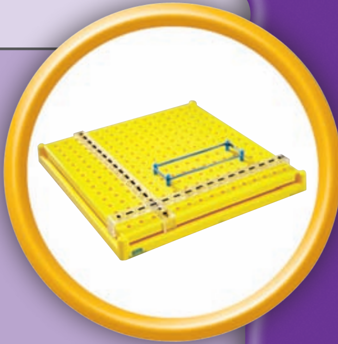
Which of the following diagrams does not show a translation?

A. 

B. 

C. 

D. 



Try It! 45 minutes | Pairs

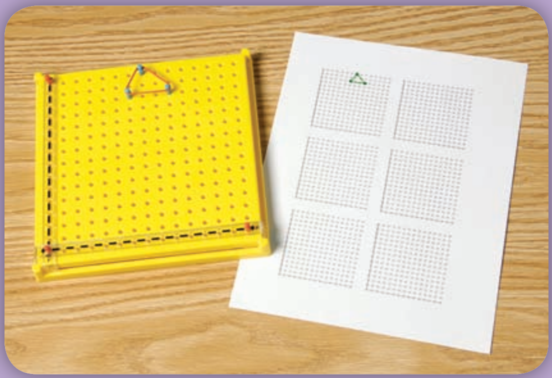
Here is a problem about translations (slides).

Marielle made a flipbook, a type of simple animation, in art class. Each drawing in the book shows the same object in a slightly different position. When the pages are flipped, the object appears to move. Marielle wants to make a flipbook that shows a triangle moving from the top center of the page to the lower right. What will her book look like?

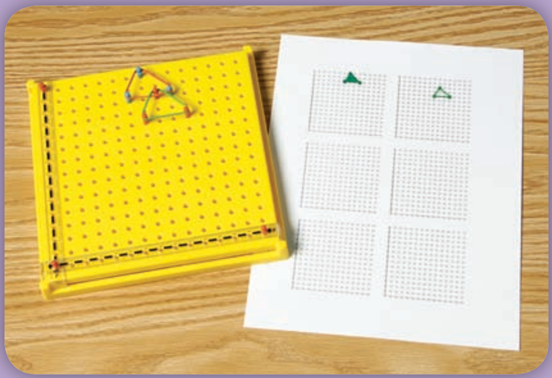
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials. Have students set up the pegboard for Quadrant I graphing.

Materials

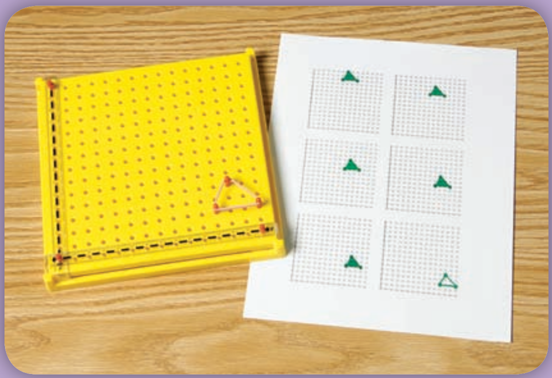
- XY Coordinate Pegboard
- Mini Dot Paper Grids (BLM 8; 1 per pair)
- card stock
- glue stick
- scissors
- marker



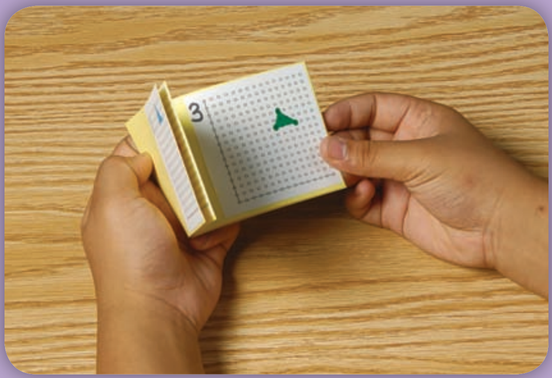
1. Have students use blue pegs to plot the following three points to form a triangle: (6, 12), (7, 14), and (9, 12). Have them connect the pegs with a rubber band. Then have them transfer the triangle to the first grid on the BLM. They may color in the triangle.



2. Have students "slide" each vertex of the triangle down 2 spaces and right 1 space. Have them use red pegs to mark the new vertices and stretch a rubber band around them. Have students remove the first triangle and then transfer the new triangle to the second grid on the BLM.



3. Repeat the procedure until all six grids have a triangle plotted on them.



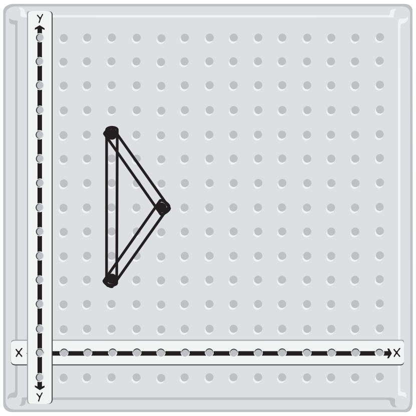
4. Have students number the grids in order, cut them out, and glue each one to the upper right-hand corner of a card. Students should stack the cards in order and flip them with their thumb to create a "traveling triangle."

Answer Key

Use an XY Coordinate Pegboard to model the triangle. Write the coordinates of the vertices. Translate the original triangle and write the coordinates of the vertices of each new triangle.

(Check students' work.)

1.



vertices of triangle (3, 3), (5, 6), (3, 9)

Slide 2 units right (5, 3), (7, 6), (5, 9)

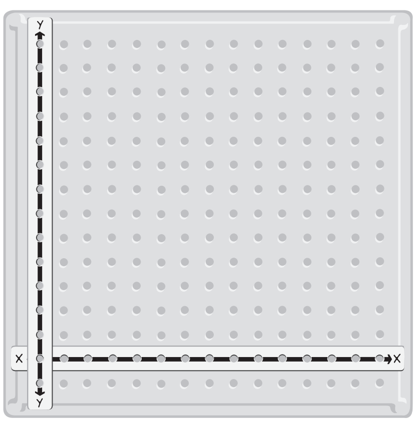
Slide 4 units right, 1 unit down (7, 2), (9, 5), (7, 8)

Slide 5 units right, 3 units up (8, 6), (10, 9), (8, 12)

Using an XY Coordinate Pegboard, model the rectangle with the given vertices. Translate the original rectangle. Sketch the translation and write the coordinates of the new rectangle.

(Check students' models.)

2. (2, 1), (2, 6), (5, 1), (5, 6)



Slide 3 units right (5, 1), (5, 6), (8, 1), (8, 6)

Slide 1 unit down (2, 0), (2, 5), (5, 0), (5, 5)

Slide 2 units right, 4 units up (4, 5), (4, 10), (7, 5), (7, 10)

Name the coordinates of each vertices after the translation described.

3. translate the triangle 4 units right

(2, 5) (6, 5)
(3, 1) (7, 1)
(5, 5) (9, 5)

4. translate the triangle 2 units left and 2 units up

(4, 0) (2, 2)
(6, 7) (4, 9)
(3, 5) (1, 7)

5. translate the triangle 6 units right and 3 units down

(7, 9) (13, 6)
(6, 10) (12, 7)
(9, 5) (15, 2)

Answer Key

Challenge! Describe how to find the new location of a point that is translated when you do not have a coordinate grid in front of you.

Challenge: (Sample) Add the right or subtract the left movement to the *x*-coordinate and add the up or subtract the down movement to the *y*-coordinate.

LESSON 4

Objective

Rotate a plane figure a given number of degrees around a fixed point.

Common Core State Standards

- **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Geometry

Rotations (Turns)

A *rotation* is a transformation that moves every point of a figure around a fixed point (usually a vertex or the origin of the coordinate plane). A rotation creates a figure that is congruent to the original figure. A rotation preserves distance from and orientation to the fixed point.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Does the rotated (turned) figure look the same as the original figure (pre-image)? How is it like the original? How is it different?*
- **Ask:** *What does it mean for a figure to maintain its orientation as it turns?*
- **Ask:** *How many degrees did you rotate the trapezoid in order to bring it back to the starting point?*
- **Ask:** *What would happen if you rotated the trapezoid 60° each time? How many trapezoids would there be in the circle you generated?*

Solve It

Reread the problem with students. Have them write a brief description (with simple sketches) of how a rotation does not affect an object's distance from and orientation to a fixed point.

More Ideas

For other ways to teach about rotations—

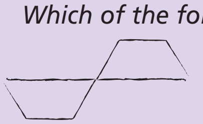
- Have the students rotate other Pattern Blocks on graph paper and notice how the figures maintain their orientation and distance regarding a center point.
- Have students practice rotations of multiples of 90° using the XY Coordinate Pegboard. Have students plot a shape and then re-peg it using clockwise and counterclockwise rotations.
- A Geoboard can be used similarly to the XY Coordinate Pegboard.

Formative Assessment

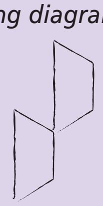
Have students try the following problem.

Which of the following diagrams shows a 90° rotation?

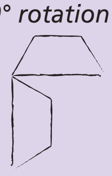
A.



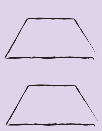
B.



C.



D.



Try It! 30 minutes | Pairs

Here is a problem about rotations (turns).

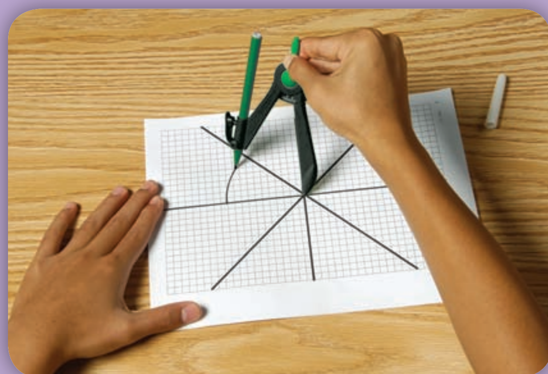
Jared is setting up stations for a relay game. The eight stations are to be evenly spaced from each other and set 10 feet from a center peg. The stations are trapezoids. Their smaller bases should face the peg. He has been asked to make a diagram showing where each station should be. What will his diagram look like?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

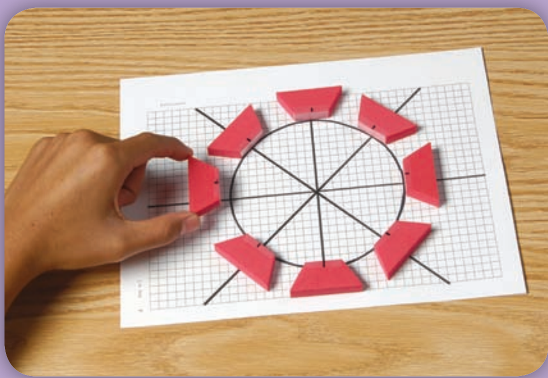
Materials

- Pattern Blocks (8 trapezoids)
- $\frac{1}{4}$-Inch Grid Paper (BLM 6; 1 per pair)
- marker
- tape
- drawing compass
- ruler
- protractor

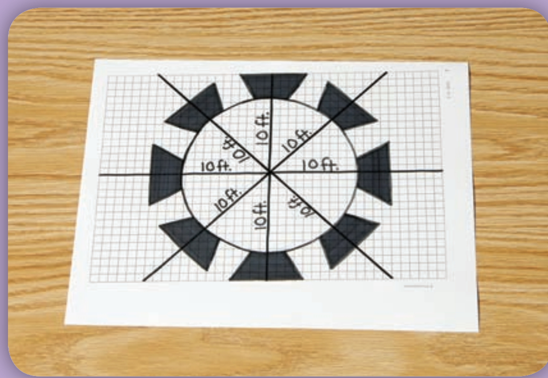
1. Have students put a piece of tape over the small base of each trapezoid. Then have them find and mark the middle of the small base.



2. Have students draw the *x*- and *y*-axes centered on the graph paper. **Ask:** *How many degrees must Jared rotate each station so that he can place them all evenly around the peg?* Next, have them draw lines that are at a 45° angle to the axes. Then have them draw a circle with a radius of 10 units from the origin.



3. Have students center a trapezoid over the point (10, 0) on the *x*-axis. **Ask:** *What is the distance from the middle of the circle to the marked point on the trapezoid?* (10 units) Have them put the remaining blocks in place where the lines intersect the circle.

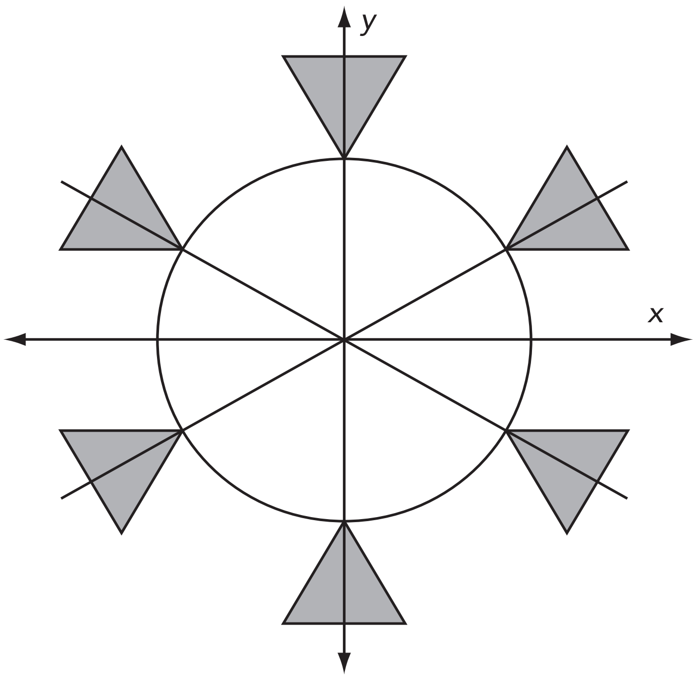


4. Finally, have students outline the trapezoids. They should convert the markings to feet using a scale of 2.5 inches:10 feet and label the length of each of the radii on the diagram.

Answer Key

Use Pattern Blocks and grid paper to model the rotations shown below. The triangle is rotated about the origin. Fill in the blanks. Find the degrees of rotation. (Check students' work.)

1.



One complete rotation is 360°.

The triangle is rotated 6 times.

The angle of rotation is 60°.

Using Pattern Blocks and grid paper, model the rotation of a square 4 times about the origin. Sketch the model.

2. (Check students' models.)

What is the angle of rotation when a square is rotated about the origin 4 times? 90°

Identify the angle of rotation about the origin for each number of rotations.

3. 5 rotations 72°

4. 12 rotations 30°

5. 3 rotations 120°

6. 10 rotations 36°

7. 9 rotations 40°

8. 15 rotations 24°

Answer Key

Challenge! Why do you use 360° as the dividend when you are finding an angle of rotation?

Challenge: (Sample) 360° is one complete rotation around a circle.

LESSON 5

Objective

Dilate a plane figure.

Common Core State Standards

- **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Geometry

Dilations

A *dilation* is a transformation that produces an image that is the same shape as the original but of a different size. Dilations with a scale factor greater than 1 produce a larger image, while scale factors between 0 and 1 produce a smaller image. Students at this level start their investigation of transformational geometry with reflections, translations, and rotations. A key point in understanding these transformations is that they result in figures that are congruent to the original. A dilation produces a figure that is similar to, but not congruent to, the original.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What points did you have to plot on the pegboard to model Lauren's pool? Lauren's patio? Lauren's yard?*
- **Ask:** *What is the scale factor of the pool to the pool and patio together?*
- **Ask:** *What is the scale factor of the pool to the yard?*
- **Ask:** *Is the area of the pool and patio together larger than one-half the area of the yard? Will the city allow Lauren to build the patio?*

Solve It

Reread the problem with students. Have students explain whether the city will allow Lauren to build the patio. They should use the terms *scale factor* and *dilation* in their explanations.

More Ideas

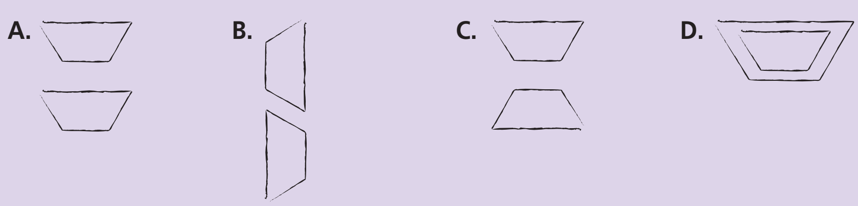
For another way to teach about dilations—

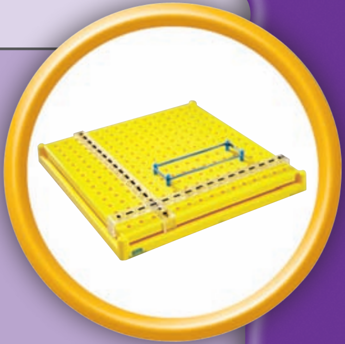
- Use Color Tiles or Centimeter Cubes to create growing (dilating) squares or rectangles with a scale factor of 2, 3, and 4.

Formative Assessment

Have students try the following problem.

Which of the following diagrams shows a dilation?

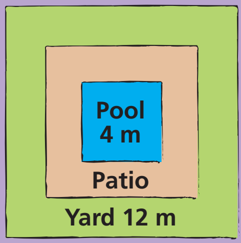




Try It! 35 minutes | Pairs

Here is a problem about dilations.

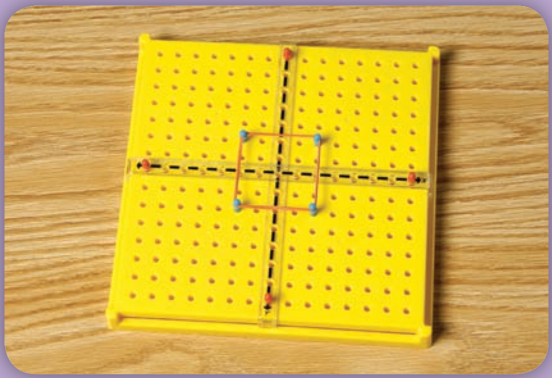
Lauren has a square pool in her backyard that is 4 meters on a side. She wants to surround it with a square patio whose sides are double the length of the pool. However, a city ordinance says that a pool and its surrounding patio cannot cover more than half the area of the yard. Her yard is a square 12 meters on a side. Will the city allow her to build the patio?



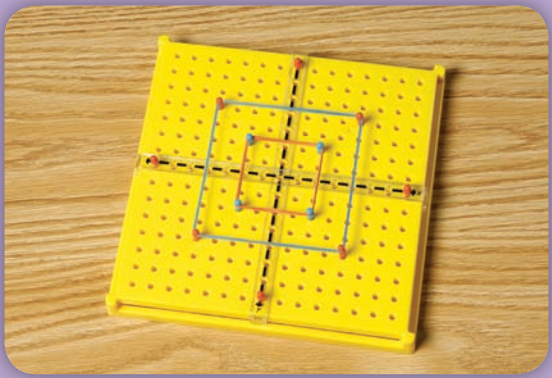
Introduce the problem. Then have students do the activity to solve the problem. Sketch the diagram on the board. Review scale factor, if necessary. Distribute the materials. Have students set up the XY Coordinate Pegboard for four-quadrant plotting.

Materials

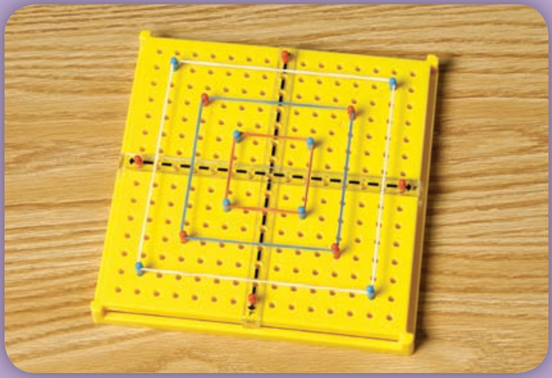
- XY Coordinate Pegboard
- paper (1 sheet per pair)
- pencils (1 per pair)



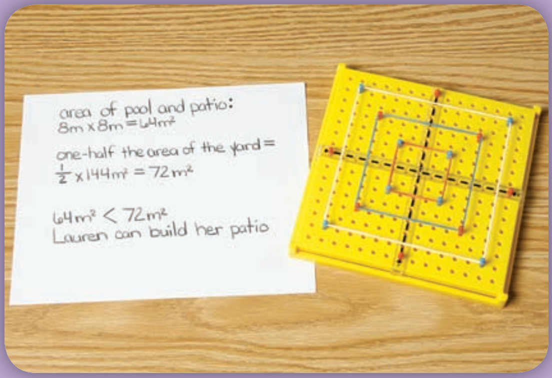
1. Have students plot the following points: (–2, 2), (2, 2), (2, –2), (–2, –2). Then have them connect the points with a rubber band to represent Lauren's pool.



2. Next have students determine the scale factor for the patio and then plot the patio on the pegboard. **Say:** *The scale factor is the number by which you multiply the original dimensions to get the new dimensions.* The scale factor is 2.



3. Now have students plot the sides of the yard on the pegboard. Students may need to use a separate rubber band for each side of this square.

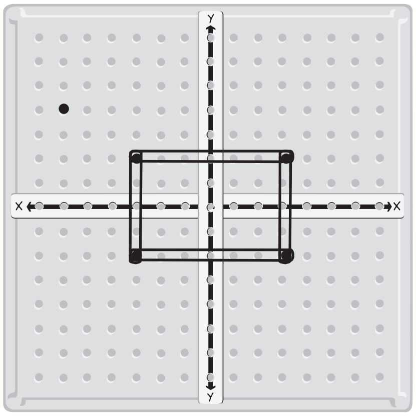


4. Have students calculate the area of the pool and patio together and compare it to the area of the yard. **Ask:** *Will the city allow Lauren to build the patio?*

Use an XY Coordinate Pegboard to model each rectangle. One vertex of a dilation is shown. Find the other vertices. Write the coordinates of the vertices after the dilation. What is the scale factor?

(Check students' work.)

1.



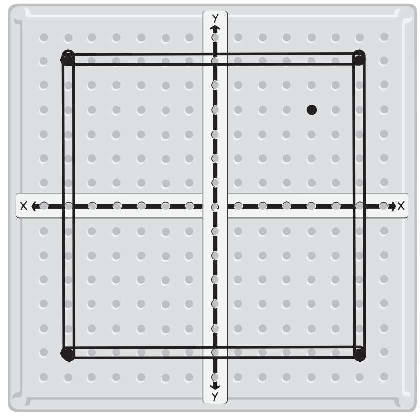
vertices of dilated image

(–6, 4); (–6, –4); (6, 4); (6, –4)

scale factor

2

2.



vertices of dilated image

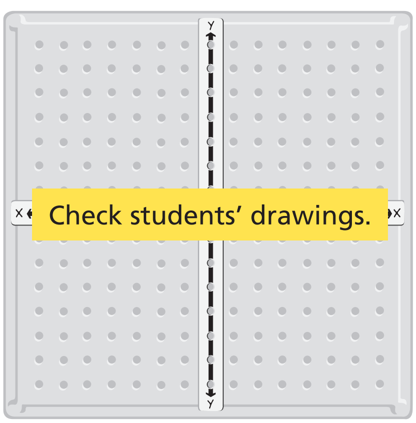
(–4, 4); (–4, –4); (4, –4); (4, 4)

scale factor

$\frac{2}{3}$

Using an XY Coordinate Pegboard, model a rectangle. Draw the figure and the dilated images on the grid below. Name the coordinates of the original rectangle and the dilated rectangle.

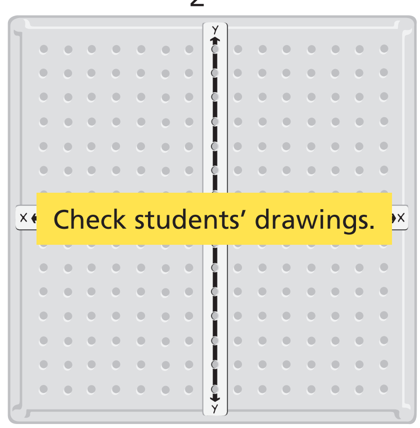
3. scale factor: 3



vertices of rectangle

vertices of dilated rectangle

4. scale factor: $\frac{1}{2}$



vertices of rectangle

vertices of dilated rectangle

Answer Key

Challenge! Explain how you know from the scale factor if the dilated figure will be an enlargement or a reduction of the original figure.

Challenge: (Sample) If the scale factor is greater than 1, the dilated figure is a bigger size than the original figure. If the scale factor is a fraction less than 1, the dilated figure is a smaller size than the original figure.

LESSON

6

Objective

Demonstrate the triangle sum theorem.

Common Core State Standards

- **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Geometry

Triangle Sum Theorem

The triangle sum theorem states that the sum of the interior angles of any triangle equals 180°. In this activity, students demonstrate the theorem, which may be applied in order to determine the third angle of a triangle when the other two angles are known.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *When you assembled the triangles so that each of the different angles met at a point, you were demonstrating, in one way, the triangle sum theorem. State this theorem in your own words.*
- **Ask:** *If you know two of a triangle's interior angles, how can you use this information and the triangle sum theorem to find the third?*
- **Ask:** *Are there other ways to assemble these three triangles and still satisfy the theorem?*
- **Ask:** *If three triangles are similar but not congruent, will you still be able to assemble them to form a straight angle? Explain.*
- **Ask:** *Can a right triangle contain an obtuse angle? Explain.*

Solve It

Reread the problem with the students. Have them write a formula that reflects what they have demonstrated with the AngLegs® triangles: In any triangle, $m\angle A + m\angle B + m\angle C = 180°$.

More Ideas

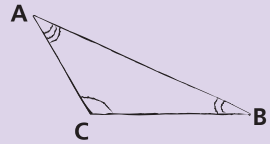
For other ways to teach about the triangle sum theorem—

- Have students confirm their findings by measuring the interior angles of each triangle carefully with a protractor. Note: Instruct students to measure from the center groove on the AngLegs pieces. Some rounding may have to be done.
- Have students assemble equilateral triangles made from various AngLegs pieces. For example, students can assemble a green equilateral triangle, a yellow equilateral triangle, and a red equilateral triangle. **Ask:** *The triangles are not congruent. Why can you still assemble them into a straight line?*

Formative Assessment

Have students try the following problem.

Find $m\angle C$ if $m\angle A$ is 35° and $m\angle B$ is 21°.



A. 56° **B.** 124°
C. 145° **D.** 304°

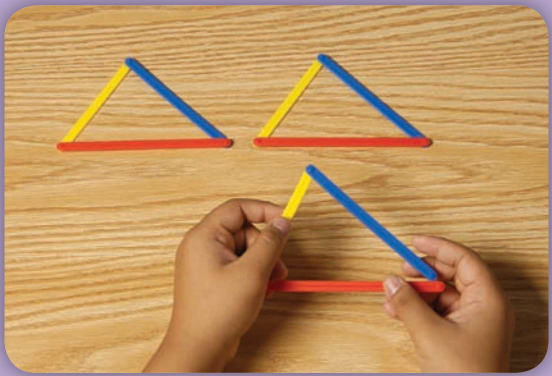
Try It! 30 minutes | Groups of 4

Here is a problem about the triangle sum theorem.

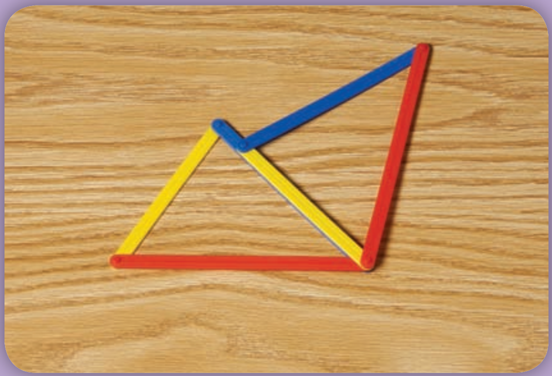
Farah works for an architectural firm that designs theme parks. Her current client wants a park with three distinct areas—one for rides, one for stage shows, and one for concessions. Each area must be triangular and identical in size and shape. These three triangular areas must touch without gaps or overlaps and form a straight line on one side, where the parking lot will be. Farah thinks she can apply the triangle sum theorem to the problem. How can Farah arrange the three areas to satisfy the client's demands?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

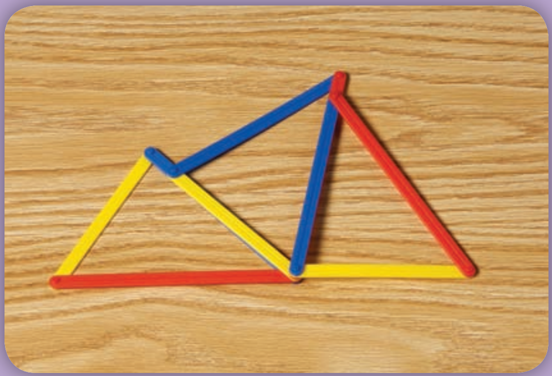
Materials
- AngLegs®



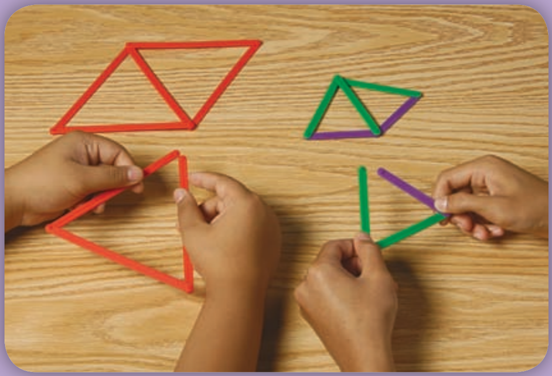
1. Have students create three identical scalene triangles using 1 blue, 1 red, and 1 yellow AngLegs piece. You may refer to the various angles in terms of the legs that form them.



2. Now have students begin to assemble the triangles so that the three different angles meet at the same point. Begin by orienting the first triangle as shown. Attach another triangle to it, matching up the blue-red angle of the original figure with the red-yellow angle in the second triangle.



3. Have students attach the third triangle by its blue-yellow angle. **Say:** *Look at the structure you just assembled from the three triangles.* **Ask:** *What do you notice about the bottom of the figure?* Elicit that it forms a straight line (straight angle). **Ask:** *How many degrees are there in a straight line or straight angle? What is the sum of the three angles in any triangle?*

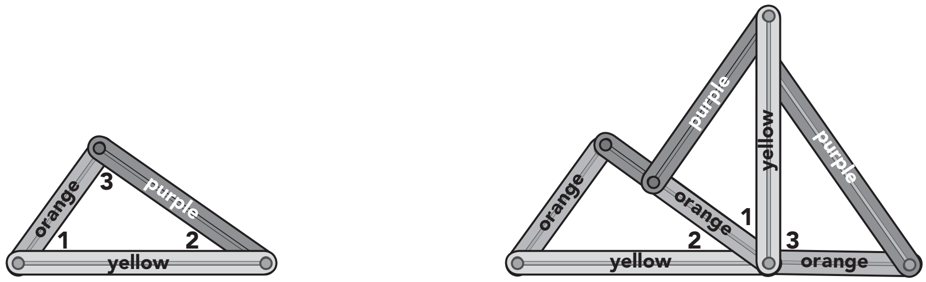


4. Have students test the triangle sum theorem, first with an isosceles triangle (2 green, 1 purple), and then with an equilateral triangle (3 red). **Ask:** *Does the theorem hold true for these types of triangles, too?*

Use AngLegs to model the triangle three times. Assemble the triangles as shown. Find the sum of the angles of the triangle. Explain your answer.

(Check students' work.)

1.



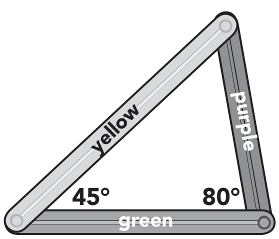
m∠1 + m∠2 + m∠3 = 180°

The three angles form a straight line, or straight angle. The measure of a straight angle is 180°.

Using AngLegs, model three triangles that match the given triangle. Assemble the triangles to form a straight line. Sketch the model. Find the measure of the unknown angle.

2.

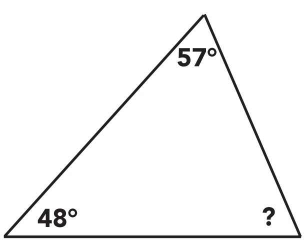
(Check students' models.)



missing angle = 55°

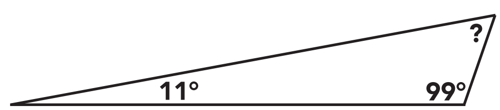
Find the measure of each unknown angle.

3.



75°

4.



70°

Two angle measures of a triangle are given. Find the third.

5. 58°, 39° 83°

6. 33°, 35° 112°

7. 106°, 22° 52°

8. 90°, 45° 45°

Answer Key

Challenge! Explain how you can model an isosceles triangle using AngLegs. What is the measure of the base angles of an isosceles triangle that has a third angle with a measure of 50°? Draw a picture to help. Show your work.

Challenge: (Sample) Use two AngLegs of the same color and one of a different color.
65°; 180 – 50 = 130; 130 ÷ 2 = 65

LESSON 7

Geometry

Parallel Lines Intersected by a Transversal

Objective

Use the properties of parallel lines cut by a transversal to find the measure of unknown angles.

Common Core State Standards

- **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

As students investigate parallel lines that are intersected by a transversal, they discover the relationships between the corresponding angles that are formed. These relationships then can be used to find the measures of unknown angles when given the measure of one of these angles.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

- **Ask:** *Which of the angles are the same? What is the measure of these angles?* Angles A, C, E, G, I, and K = 60°. B, D, F, H, J, and L = 120°.
- **Say:** *Look at a parallelogram formed by the handrail, the baserail, and two balusters.* **Ask:** *What is the relationship between the angles that have the same measure (E and G, for example)?* Elicit that they are opposite (in opposite corners from) each other.
- **Say:** *Look at all three parallelograms.* **Ask:** *What do angles A, E, and I have in common?* Elicit that they are in corresponding positions in the parallelograms. Have students find other examples.
- **Say:** *Look at angles E and B. They are formed by the handrail and a baluster.* **Ask:** *What is the sum of their angles?* **Say:** *Find other examples.*
- **Say:** *Look at angles J and L. Think of them as alternate interior angles.* **Ask:** *What is their relationship?* **Say:** *Find other examples.*

Solve It

Reread the problem with students. Have students explain in writing how to find all of the angles formed by a transversal line that crosses parallel lines when the measure of only one of the angles is given.

More Ideas

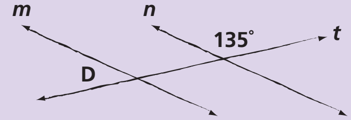
For other ways to teach about parallel lines intersected by a transversal—

- Use AngLegs® pieces to construct other configurations that have parallel lines intersected by a transversal.
- Have students look for examples of parallel lines intersected by a transversal in architecture and art.

Formative Assessment

Have students try the following problem.

Lines m and n are parallel. What is the measure of angle D?

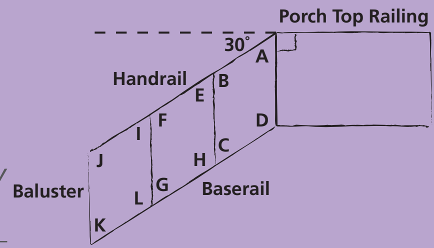


A. 40° **B.** 45° **C.** 90° **D.** 135°

Try It! 30 minutes | Pairs

Here is a problem about parallel lines intersected by a transversal.

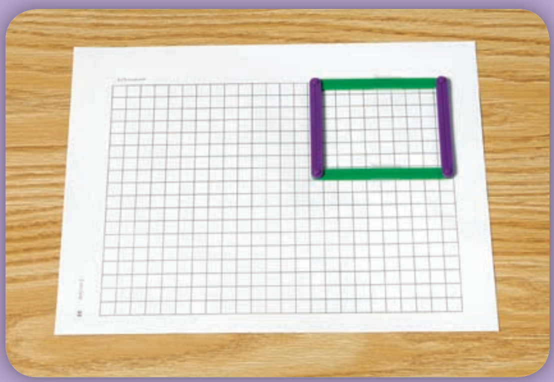
Adam is drawing plans to build a handrail for the stairs to the back porch. He knows that the handrail should slant down 30° from the top railing of the porch. What angles will the balusters form with the new handrail and with the baserail?



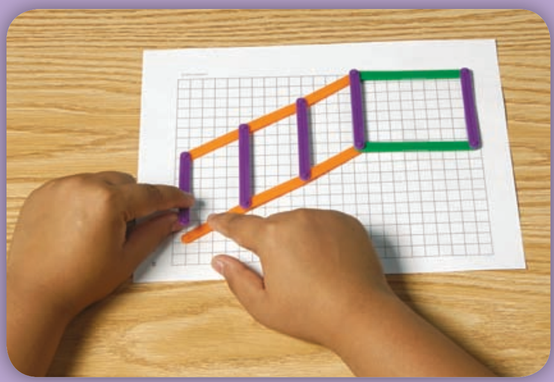
Introduce the problem. The have students do the activity to solve the problem. Sketch the diagram from the story problem on the board. Distribute the materials. Remind students to measure from the groove down the middle of the AngLegs piece to get the correct reading.

Materials

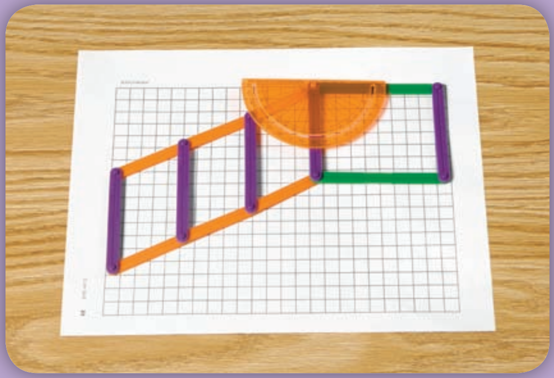
- AngLegs® (6 orange, 5 purple, and 2 green)
- Centimeter Grid Paper (BLM 9; 1 per pair)
- tape
- paper
- ruler or straightedge



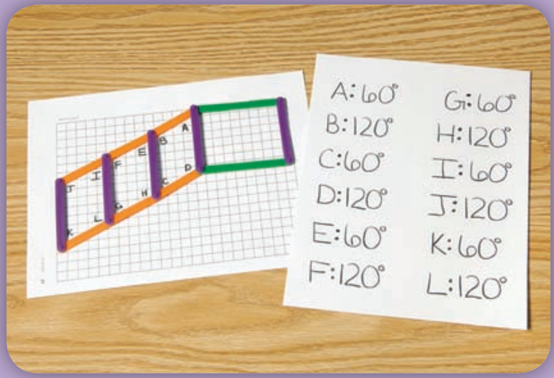
1. Have students construct the porch model with two green and two purple AngLegs pieces as shown. Place the model in the upper right-hand corner of the graph paper. Use the grid to make sure that the corners are square (90°) and tape the figure in place.



2. Have students use three orange AngLegs pieces to make the top rail (handrail) and three to make the bottom (baserail). Have them use purple AngLegs pieces for the balusters.



3. Have students use a protractor to set the handrail assembly at a 30° angle from the top porch railing. The purple balusters should be set at 90° (vertical). Students should secure the model to the grid paper with tape as needed.

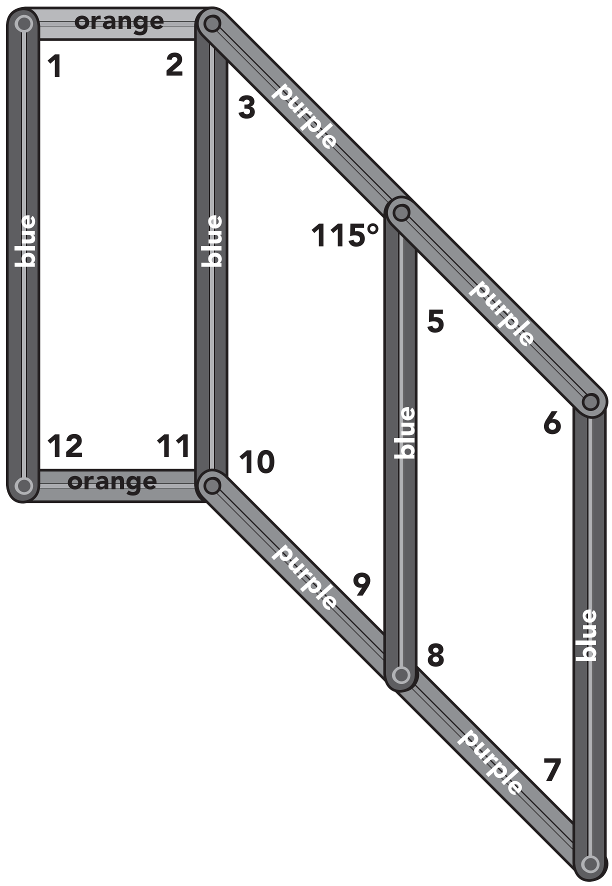


4. Have students examine their model and measure each angle that is labeled in the drawing that accompanies the story problem. Have them record their answers on a separate piece of paper.

Answer Key

Use AngLegs to build the model shown. Use the AngLegs protractor to measure each angle. Which angles are congruent? (Check students' work.)

1.



m∠1 = 90°

m∠2 = 90°

m∠3 = 65°

m∠4 = 115°

m∠5 = 65°

m∠6 = 115°

m∠7 = 65°

m∠8 = 115°

m∠9 = 65°

m∠10 = 115°

m∠11 = 90°

m∠12 = 90°

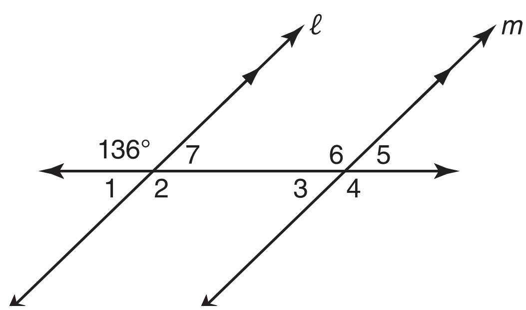
Angles congruent to ∠1: 2, 11, and 12

Angles congruent to ∠3: 5, 7, and 9

Angles congruent to ∠4: 6, 8, and 10

Use the measure given to find the measures of the other angles. $\ell \parallel m$.

2.



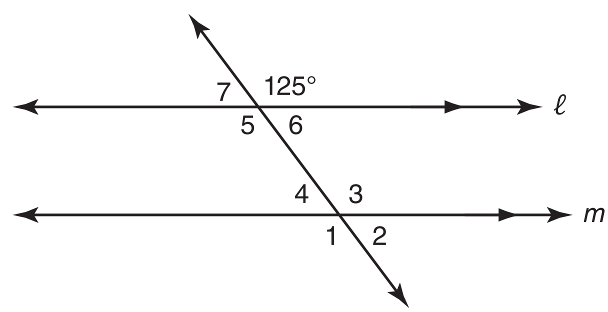
m∠1 = 44° m∠2 = 136°

m∠3 = 44° m∠4 = 136°

m∠5 = 44° m∠6 = 136°

m∠7 = 44°

3.



m∠1 = 125° m∠2 = 55°

m∠3 = 125° m∠4 = 55°

m∠5 = 125° m∠6 = 55°

m∠7 = 55°

Answer Key

Challenge! When given two parallel lines cut by a transversal, if you are given the measure of one angle, how do you find the measures of the other seven angles? Draw a diagram to help.

Challenge: (Sample) There are three angles congruent to the given angle. The other four angles are supplementary to the given angle.

LESSON 8

Geometry

Pythagorean Theorem

Students at this level can be expected to develop a visual understanding of the Pythagorean theorem: In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse ($a^2 + b^2 = c^2$). Application of the Pythagorean theorem allows students to develop skills in indirect measurement.

Objective

Develop a visual understanding of the Pythagorean theorem.

Common Core State Standards

- **8.G.6** Explain a proof of the Pythagorean Theorem and its converse.
- **8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What number sentences can we write that will show what we just did with the tiles?* ($4^2 + 3^2 = 25$) *What size square did you build with the 25 tiles?* (5 by 5) *What is the square root of 25?*
- **Say:** *The numbers 3, 4, and 5 form what is called a Pythagorean triple. These are three whole numbers that satisfy the Pythagorean theorem.*
- **Ask:** *If we double the lengths of the legs of the triangle (i.e., 6, 8, 10), will the relationship still hold? Explain.*

Solve It

Reread the problem with students. Have them explain how the Pythagorean theorem can be applied to help Jack and his father determine the length of the ramp they will need to build. The explanation should include sketches.

More Ideas

For another way to teach about the Pythagorean theorem—

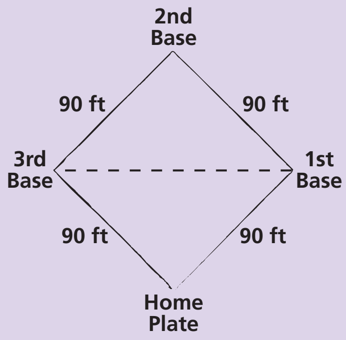
- Use Color Tiles to teach students the concept of estimating square roots that are not integral values. Use the tiles to help students conceptualize perfect squares and then use that to determine that the square root of 21 is between 4 and 5.

Formative Assessment

Have students try the following problem.

A baseball diamond is a square, 90 feet on a side. What is the approximate distance the ball travels when the first baseman throws the ball from first base to third base?

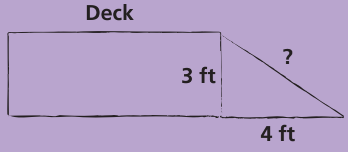
A. 90 feet　　**B.** 127 feet
C. 180 feet　　**D.** 360 feet



Try It! 20 minutes | Pairs

Here is a problem about the Pythagorean theorem.

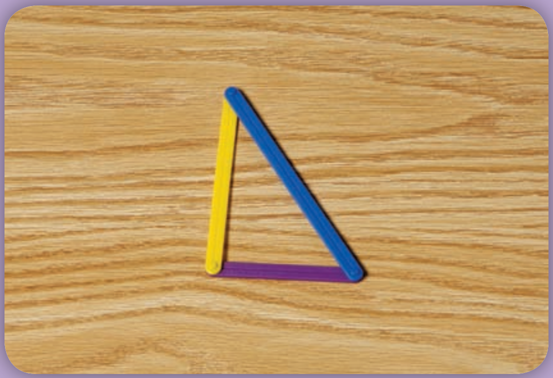
Jack's parents just bought a new washer and dryer, and he wants to help his father set up a ramp to get them into the house. The deck that leads into the back door is 3 feet high, and the sidewalk ends 4 feet from where the deck starts. How long does the ramp need to be?



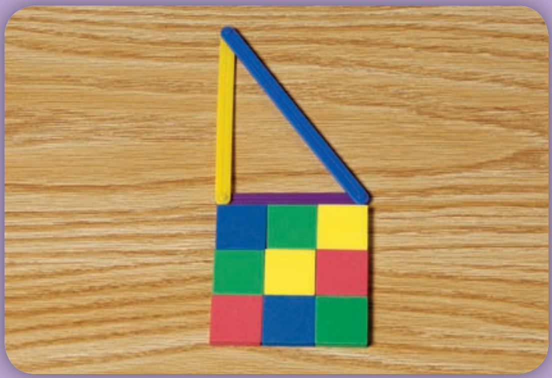
Introduce the problem. Then have students do the activity to solve the problem. Sketch the diagram from the story problem on the board. Distribute the materials.

Materials

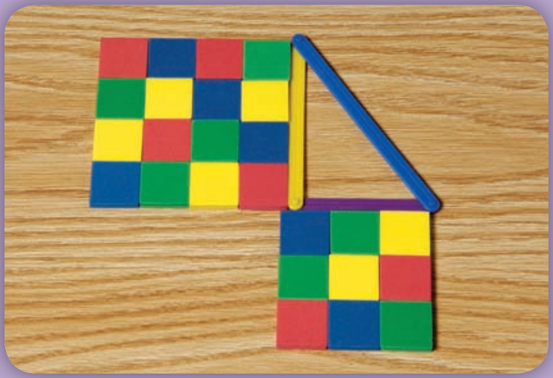
- AngLegs® (1 purple, 1 yellow, and 1 blue)
- Color Tiles (at least 25 per pair)



1. Have students form a right triangle using the purple, yellow, and blue AngLegs pieces. Remind students that the purple and yellow sides of the right triangle are called *legs* and that the blue side is called the *hypotenuse.*



2. Have students place three tiles along the length of the purple AngLegs piece. Then have them complete the 3 by 3 square (a total of 9 tiles) on that leg.



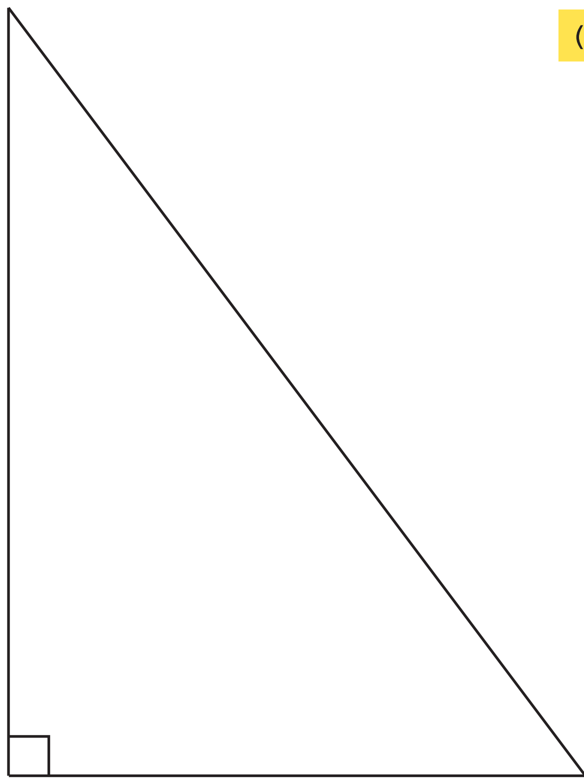
3. Have students place 4 tiles along the length of the yellow AngLegs piece. Then have them complete the 4 by 4 square (a total of 16 tiles).

4. Have students take all of the tiles from the purple and yellow legs and recombine them into a square along the hypotenuse of the triangle. Have them revisit the story problem and determine the length of the ramp using the relationship $a^2 + b^2 = c^2$.

Use Color Tiles to model a square the length of the hypotenuse of the triangle shown. Use those tiles to build squares on the legs of the triangle. Write the equation for the Pythagorean theorem.

1.

(Check students' work.)



$9 + 16 = 25$

$3^2 + 4^2 = 5^2$

Find another Pythagorean triple and illustrate that it satisfies the Pythagorean theorem. Sketch the model. Write the equation.

2. Check students' models and equations.

Show the equation you can use to determine if each set of numbers can be the lengths of sides of a right triangle. Write yes or no.

3. 5, 12, 13

$25 + 144 = 169$; yes

4. 4, 7, 8

$16 + 49 \neq 64$; no

5. 12, 16, 20

$144 + 256 = 400$; yes

Answer Key

Challenge! Explain how to test three numbers to see if they could be the lengths of the sides of a right triangle. What number will represent the hypotenuse? Explain.

Challenge: (Sample) Substitute values for *a*, *b*, and *c*. Square the numbers and check that the equation is true. The greatest number will represent the hypotenuse because the right angle is the largest angle and the longest side is always opposite the largest angle.