

LINEAR ALGEBRA

LECTURE 1: VECTOR SPACES

1.1. DEFINITION. A vector space V over a field F is a set V with two binary operations, addition $+$ and scalar multiplication \cdot , satisfying the following axioms:

- $(V, +)$ is an abelian group.
- (F, \cdot) is a field.
- $a(bv) = (ab)v$ for all $a, b \in F$ and $v \in V$.
- $(a+b)v = av + bv$ for all $a, b \in F$ and $v \in V$.
- $a(v+w) = av + aw$ for all $a \in F$ and $v, w \in V$.
- $1v = v$ for all $v \in V$.



1.2. PROPOSITION. Let V be a vector space over a field F . Then the zero vector 0 is the unique element in V such that $0 + v = v$ for all $v \in V$. Similarly, the additive inverse of v is the unique element $-v$ in V such that $v + (-v) = 0$.