

EXERCISE 2.5.8

Prove that \mathbb{R}^n is a vector space.

Solution. We first show that \mathbb{R}^n is an abelian group under addition. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Then

$$\begin{aligned} \mathbf{u} + \mathbf{v} &= (u_1, \dots, u_n) + (v_1, \dots, v_n) \\ &= (u_1 + v_1, \dots, u_n + v_n) \\ &= (v_1 + u_1, \dots, v_n + u_n) \\ &= (v_1, \dots, v_n) + (u_1, \dots, u_n) \\ &= \mathbf{v} + \mathbf{u}. \end{aligned}$$

Thus, \mathbb{R}^n is an abelian group.

Next, we show that \mathbb{R}^n is a vector space.

$$\begin{aligned} c(\mathbf{u} + \mathbf{v}) &= c(u_1 + v_1, \dots, u_n + v_n) \\ &= (cu_1 + cv_1, \dots, cu_n + cv_n) \\ &= (cu_1, \dots, cu_n) + (cv_1, \dots, cv_n) \\ &= c\mathbf{u} + c\mathbf{v}. \end{aligned}$$

Thus, \mathbb{R}^n is a vector space.