The Number System

In seventh grade, students develop greater understanding of the number system by exploring **rational numbers**: integers, fractions, decimals, and percents.

At this level, students apply and extend previously learned concepts of addition, subtraction, multiplication, and division to adding, subtracting, multiplying, and dividing with any of the rational numbers. For instance, students may find the sum –10 + 5 by locating –10 on a number line, moving 5 spaces in the positive direction, and interpreting the number they land on (–5) as the sum. Similarly, students apply long-division concepts to learn the difference between terminating and repeating decimals–a concept necessary for later work with rational and irrational numbers.

Students continue building on their previous work with the order of operations to solve problems with rational numbers. The **order of operations** is a set of rules for determining the order in which the operations in an expression are performed. Students will apply their expanded view of the rational numbers when they work with algebraic expressions. They will learn that rewriting an expression in different forms can help them solve problems. They will also learn to work more confidently with negative numbers by viewing and experiencing them in everyday contexts.

The Grade 7 Common Core State Standards for The Number System specify that students should–

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

The following hands-on activities enable teachers to provide rich opportunities for students to deepen their understanding of the number system, with particular regard to the rational numbers. The experiences will help students to develop a unified understanding of number–that is, to work flexibly with integers, fractions, decimals, and percents.

The Number System

Contents

LESSON 1

The Number System

Add Integers I

Addition is the same for whole numbers and integers—the grouping together of quantities. Modeling with different colors helps students to perform operations with positive and negative numbers. As students learn the rules for working with integers, they should make the connection between the models they build and the rules so that manipulating integers is not arbitrary.

Objective

Add integers.

Common Core State Standards

- **7.NS.1b** Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *After you model the problem with Two-Color Counters, why do you rearrange them to form red-yellow pairs? What does a red-yellow pair represent in terms of yardage?* Students should recognize that one yard lost (red) plus one yard gained (yellow) is a net change of zero, so a red-yellow pair represents no gain or loss.
- **Ask:** *Is the sum of two negative numbers always negative? Model an example to justify your answer.*

Solve It

Reread the problem with students. Since 12 > 5, the team gained more yards than it lost. Since 12 – 5 = 7, they gained 7 more yards than they lost. So their net yardage is 7 yards. Have students write –5 + 12 = 7 and explain how this equation relates to the problem.

More Ideas

For other ways to teach about adding integers—

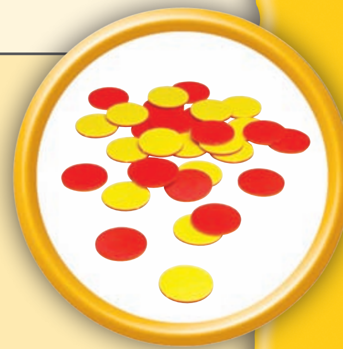
- Have students use Centimeter Cubes and a 1-cm Number Line (BLM 3) to add pairs of integers—two positive numbers: 2 + 6, two negative numbers: –1 + (–5), and a positive number and negative number: –9 + 2 or 8 + (–4). Suggest that students use red cubes for negative numbers and yellow cubes for positive numbers.
- Ask students to write their own number sentences with positive and negative numbers and use Color Tiles to model the sentences and show that a positive number plus a negative number can be positive, negative, or zero.

Formative Assessment

Have students try the following problem.

The morning temperature of –9°F is expected to rise 10 degrees by noon. What is the expected noon temperature?

A. –19°F **B.** –1°F **C.** 1°F **D.** 19°F



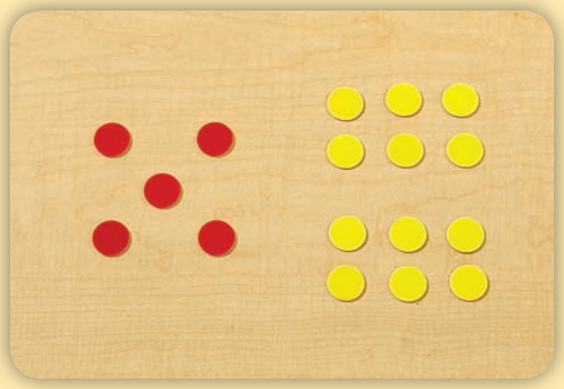
Try It! 15 minutes | Pairs

Here is a problem about adding integers.

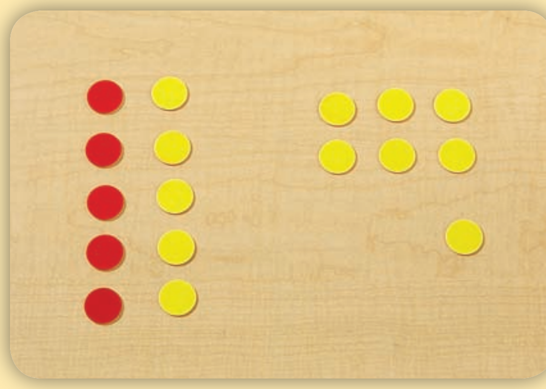
A football team lost 5 yards on one play and then gained 12 yards on the next play. What was the team's net yardage on the two plays?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

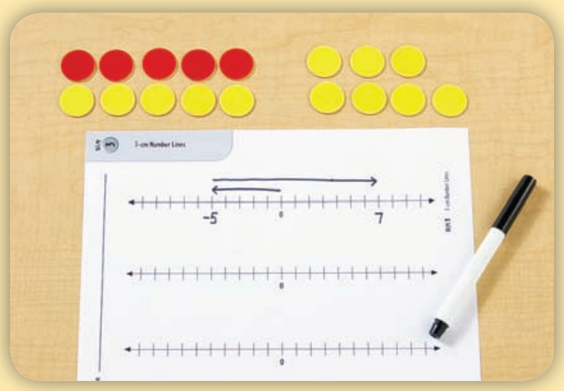
Materials
- Two-Color Counters (at least 20 per pair)
- BLM 3



1. Say: *Each red counter represents one yard lost. Each yellow counter represents one yard gained. Place counters to model this problem.* Students place 5 red counters and 12 yellow counters.



2. Say: *Move counters so that each red counter is paired with a yellow counter.* **Ask:** *What number does each red-yellow pair represent? How many yellow counters are left?* Students form 5 pairs, each representing 0. There are 7 yellow counters left, representing a net gain of 7 yards.



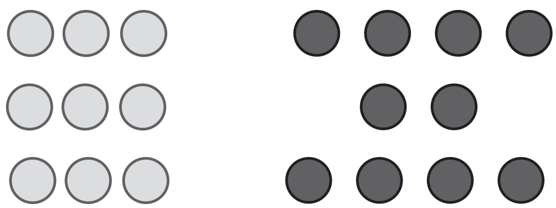
3. Say: *Now use a number line to solve this problem. Starting at 0, draw a segment 5 units to the left. From –5, draw a segment 12 units to the right.* **Ask:** *At what number do you end?* Students draw the two segments and end at 7. Help students recognize that the overlapping parts of the lines are equivalent to red-yellow pairs of counters.

⚠ Look Out!

Students may confuse negative signs with minus signs. Have students write the problem –5 + 12 = 7. Then speak the correct words: *negative five* (not *minus five*) *plus twelve equals seven.* On a number line, show students that –5 and 5 are opposites. They are both 5 units from 0, but in opposite directions.

Use Two-Color Counters to model each addition problem. Make pairs of red and yellow counters. Find the sum. (Check students' work.)

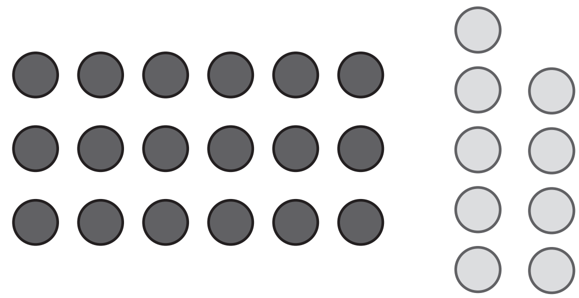
1.



9 + (–10)

–1

2.



–18 + 9

–9

Using Two-Color Counters, model each addition problem. Sketch the model. Find the sum.

3. 7 + (–4)

3

4. –12 + (–3)

–15

Find each sum.

5. 11 + (–6) 5

6. –5 + (–18) –23

7. –4 + 13 9

8. 9 + (–21) –12

9. –6 + (–14) –20

10. –3 + (–18) –21

11. 15 + 9 24

12. 26 + (–50) –24

Answer Key

Challenge! Explain how to add two integers if one is a negative number and one is a positive number. When will the sum be negative? When will the sum be positive?

Challenge: (Sample) You actually subtract the numbers, ignoring the signs, and the answer will have the same sign as the number with the greater value (ignoring the negative signs).

LESSON 2

Add Integers II

Objective

Add integers.

Common Core State Standards

- **7.NS.1b** Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Students may be familiar with integer concepts in everyday situations such as temperatures above and below zero, altitudes above and below sea level, and football yardage gained and lost. Students should recognize zero pairs and be able to use the identity property of addition to simplify computation. At the 7th and 8th grade levels, they need to be able to compute with integers in preparation for using integers in solving equations and inequalities.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the opposite of turning on a light? What is the opposite of sitting down? What is the opposite of +1? If we add +1 and its opposite (–1) what do we get?*
- **Ask:** *What does a discount do to the price of an item? How should we represent a discount coupon?*
- **Ask:** *How many zero pairs are there? After you have taken the zero pairs off the mat, what is left on the mat?*

Solve It

Reread the problem with the students. Have them write an explanation of the term "zero pairs." Students should include a discussion of why zero pairs are important in solving the problem.

More Ideas

For other ways to teach the addition of integers—

- Have pairs of students using Two-Color Counters designate one color as positive and the other color as negative. The first student makes up an addition problem with at least one negative addend and the other student models and solves it. Have students trade roles and repeat.
- Suggest that some students redo the Try It! activity with Two-Color Counters.

Formative Assessment

Have students try the following problem.

At 8:00 A.M., the temperature at Arctic base camp was –38°C. By 11:00 A.M. the temperature had risen 17 degrees. What was the 11:00 A.M. temperature?

A. –55°C **B.** –45°C **C.** –21°C **D.** 55°C

Try It! 15 minutes | Pairs

Here is a problem about adding integers.

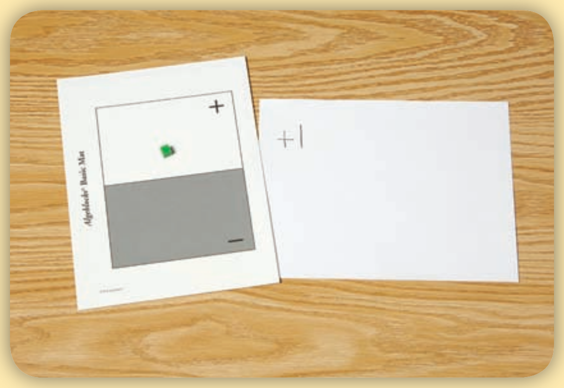
At the school store, students can use discount coupons earned in class for excellent work and good behavior. The transactions shown in the chart took place today.
How much money did the school store receive today?

Student	Purchase	Price	Coupon
Sally	Pack of Pencils	$1.00	No
Damon	Spiral Notebook	$2.00	$1.00
Rongita	Pack of Pencils	$1.00	$1.00
Chung	Protractor Set	$1.00	No
Dean	Compass Set	$2.00	$1.00
Paul	Glue Stick	$1.00	No

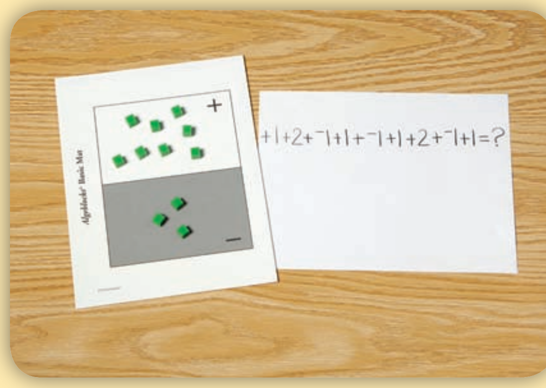
Introduce the problem. Then have students do the activity to solve the problem. Write the chart of transactions on the board. Distribute the materials.

Materials

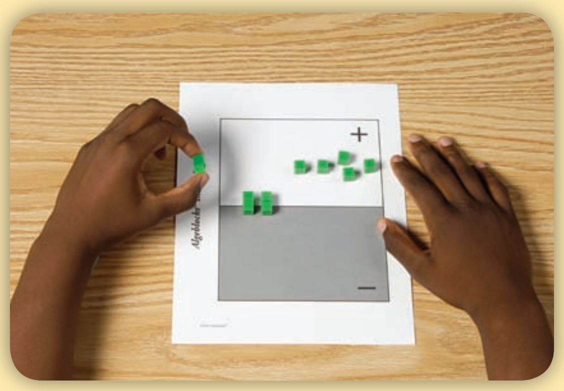
- Algeblocks® units
- BLM 4



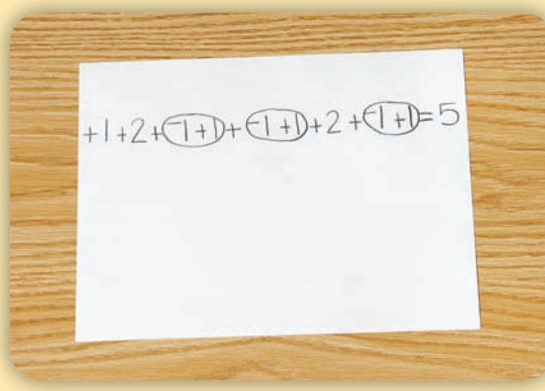
1. Have students read the first transaction from the chart. **Say:** *Put one unit block in the positive section of your Algeblocks Basic Mat. Write +1 on your paper.*



2. Have students read the second transaction. **Say:** *Put two unit blocks in the positive section of your mat, and put one unit block in the negative section. Write +2 and –1 on your paper. Now represent the rest of the transactions.*



3. Say: *Make zero pairs by pairing a unit block in the positive section with a unit block in the negative section. Repeat until all pairs are made. Now count the remaining unit blocks.*



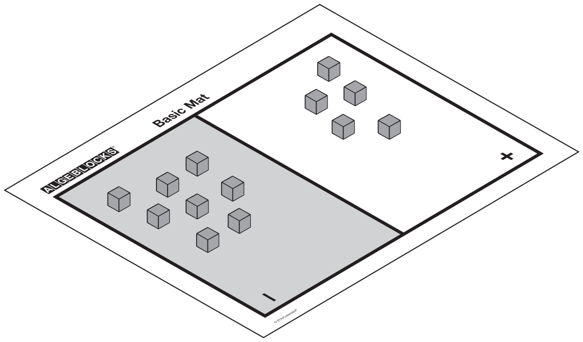
4. Have students circle all of the zero pairs. **Say:** *Add up the remaining numbers. What is the sum? This is the total amount the school store received.*

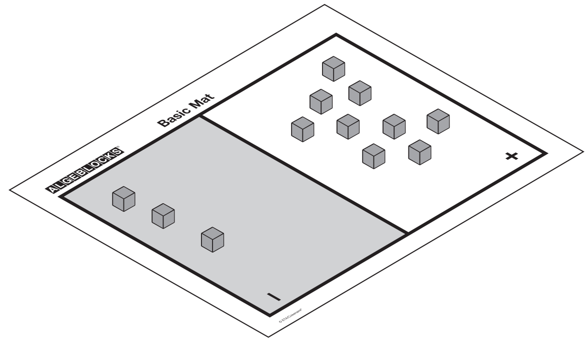
Answer Key

Use Algeblocks unit blocks and a Basic Mat to model each integer addition sentence. Make zero pairs. Write the sum. (Check students' work.)

1. 5 + (–8) = –3

2. –3 + 9 = 6

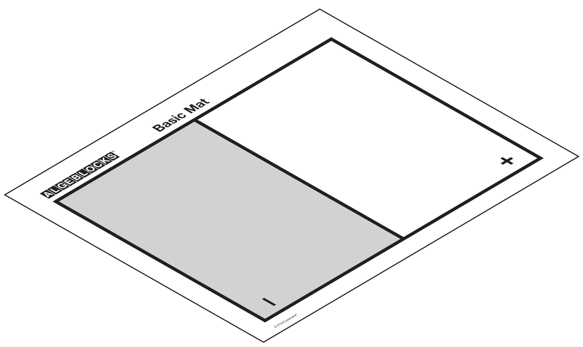


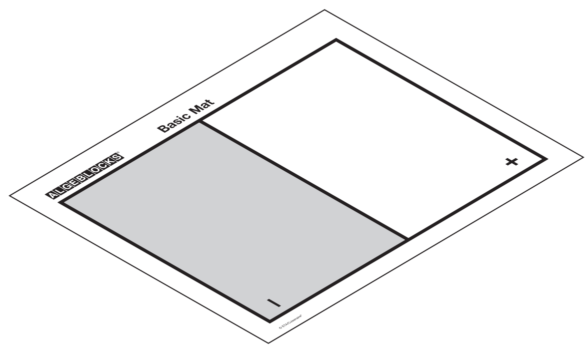


Using Algeblocks unit blocks and a Basic Mat, model each addition sentence. Sketch the model. Circle zero pairs. Write the sum.

3. 12 + (–7) = 5

4. –2 + (–5) = –7





Find each sum.

5. –5 + (–2) = –7

6. 15 + (–4) = 11

7. 8 + (–11) = –3

8. –9 + 13 = 4

9. –17 + (–4) = –21

10. 15 + 3 = 18

11. –12 + 12 = 0

12. 21 + (–7) = 14

Answer Key

Challenge! Describe how to find a sum of two integers when the signs of the integers are different. How do you decide the sign of the sum?

Challenge: (Sample) When the signs of the integers are the same, add the digits and use the sign of the numbers as the sign of the sum. When the signs are different, subtract the digits and use the sign of the digit with the greater value as the sign of the sum.

LESSON 3

Subtract Integers I

Objective

Subtract integers.

Common Core State Standards

- **7.NS.1c** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

After students show confidence with adding integers, they can learn to subtract integers. They will continue to use and develop their understanding of addition and subtraction as inverse operations. Previous work with fact families will help students to think flexibly as they add and subtract positive and negative numbers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Is this problem about a subtraction problem or an addition problem?* Discuss with students.
- **Ask:** *When you think about this problem in two different ways—as addition and as subtraction—do you get two different answers?*
- Have students write the two number sentences for this problem.

Solve It

Reread the problem with students. Notice that only red counters are used to solve the subtraction problem, $-8 - (-6) = -2$. Both yellow and red counters are used to solve the addition problem, $-8 + 6 = -2$. Make sure students understand both ways to think about this problem. Either way, Hannah still owes Rachel $2 at the end.

More Ideas

For other ways to teach about subtracting integers—

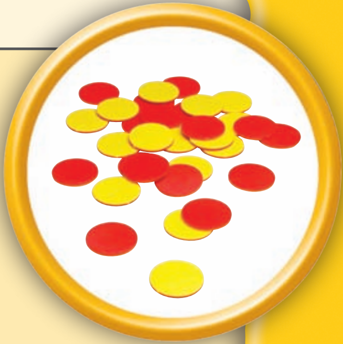
- Students can use red and yellow Color Tiles to model the problem.
- Have students use Centimeter Cubes to find $-2 - (-5)$. Suggest that students use red cubes for negative numbers and yellow cubes for positive numbers. They start with 2 red cubes and need to take away 5. But there are only 2 cubes available to take away, so 3 red-yellow pairs (which equal 0) can be added. Then 5 red cubes are removed, and 3 yellow cubes are left.

Formative Assessment

Have students try the following problem.

The current temperature is –6°F and is expected to drop 10 degrees overnight. What is the expected low temperature overnight?

A. –16°F **B.** –10°F **C.** –4°F **D.** 4°F



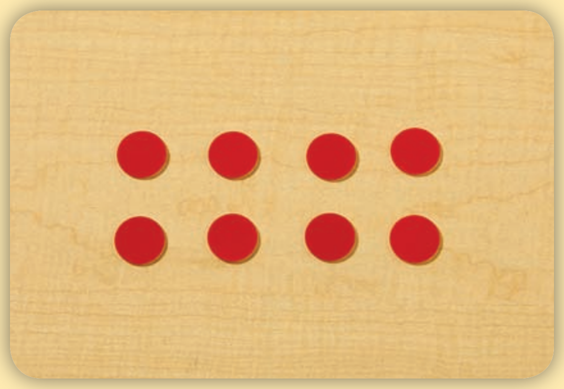
Try It!
15 Minutes | Pairs

Here is a problem about subtracting integers.

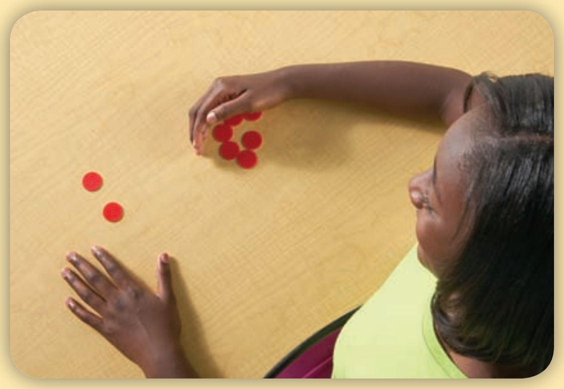
At the bookstore, Hannah borrowed $8 from her sister Rachel. At the waterpark a few days later, Rachel borrowed $6 from Hannah. What is Hannah's standing with Rachel now? Does Hannah still owe Rachel any money?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

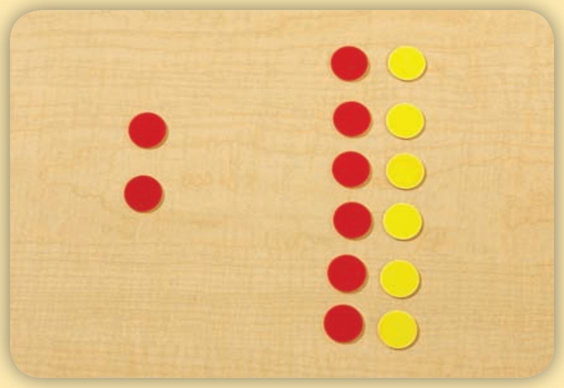
Materials
- Two-Color Counters (at least 20 per pair)



1. Say: *Let each red counter represent one dollar owed, or –1. Use counters to show Hannah's situation after borrowing $8 from Rachel.* Students place 8 red counters on the table.



2. Say: *Later, Rachel borrowed $6 from Hannah. One way to think of this is that $6 of Hannah's debt to Rachel is taken away. This is a subtraction problem: –8 – (–6). Show this with the counters.* Students take away 6 red counters, and 2 are left.



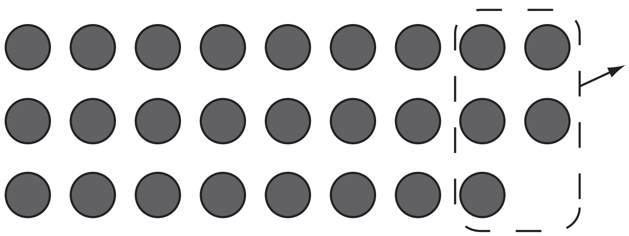
3. Say: *You can also think that when Rachel borrowed $6 from Hannah, it was the same as Hannah paying $6 back to Rachel. It is an addition problem: –8 + 6. Show this with the counters.* Students place 8 red counters, then add 6 yellow counters. They form 6 red-yellow pairs, and 2 red counters are left.

⚠ Look Out!

Students often get confused when they try to subtract a negative number, as in –8 – (–6). When they *take away* 6 red counters from a set of 8, students see that they can actually subtract a negative number. In this activity they also see that subtracting negative 6 is the same as adding positive 6: –8 – (–6) = –8 + 6. Once students are convinced of this, encourage them to use this concept whenever they see a minus sign and a negative sign together. For example, 1 – (–4) = 1 + 4 = 5.

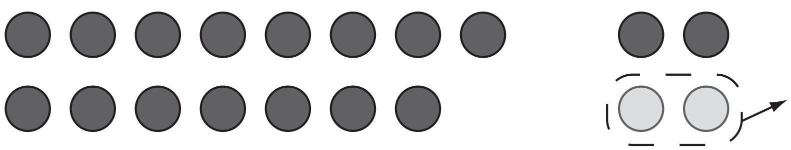
Use Two-Color Counters to model each subtraction problem. Write the number sentence for the difference. (Check students' work.)

1.



$-26 - (-5) = -21$

2.



$-15 - 2 = -17$

Using Two-Color Counters, model each subtraction problem. Sketch the model. Find the difference.

3. $9 - (-4)$ 13

4. $-14 - 5$ −19

Find each difference.

5. $21 - (-6)$ 27

6. $-15 - 7$ −22

7. $-4 - 12$ −16

8. $-9 - (-7)$ −2

Answer Key

Challenge! Rewrite Questions 5–8 as addition problems. Find the sum. Did your answers change? Explain.

Challenge: (Sample) 21 + 6 = 27; –15 + (–7) = –22; –4 + (–12) = –16; –9 + 7 = –2; No; Adding the opposite of a number is the same as subtracting a number.

LESSON 4

Objective

Subtract integers.

Common Core State Standards

- **7.NS.1c** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Subtract Integers II

The addition of integers is straightforward and fairly easy for students to perform once they understand how to use the identity property of addition to make zero pairs. Now students add zero pairs to allow them to subtract quantities that result in a remainder of less than zero. Some students may recognize that this technique is similar to the renaming that they did when they learned to subtract a larger digit from a smaller digit in second or third grade.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What does "Dropped 5°" mean in this context? Can you take away 5 unit blocks?*
- **Ask:** *Can you take away 3 unit blocks? How many zero pairs would we need to add until we have the 3 unit blocks we need to take away?*
- **Ask:** *Did you all end up with the correct noon reading of –5°? How many zero pairs did you have to add for the final temperature change?*

Solve It

Reread the problem with students. Ask the students to make a chart similar to the one in the story problem and repeat the entire process, recording the temperature changes as they work through the activity.

More Ideas

For another way to teach about subtraction of integers—

- Have pairs of students use polyhedral dice as number generators to create integer subtraction problems. The first student designates both dice as positive or as negative or one die as positive and the other as negative. The first student rolls the dice and writes a subtraction problem using the numbers thrown. The second student models the problem on his or her Algeblocks® Basic Mat and gives the answer. The first student checks the answer. Have students trade roles and repeat.

Formative Assessment

Have students try the following problem involving subtraction of integers.

The early morning temperature reading was 9°C. By noon, the temperature had dropped 15°. What was the temperature at noon?

A. –24°C **B.** –6°C **C.** 6°C **D.** 24°C

Try It! 15 minutes | Pairs

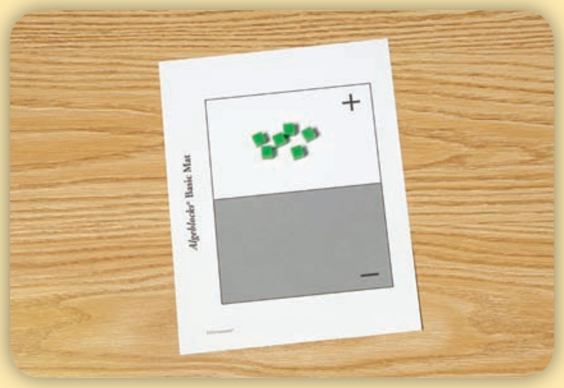
Here is a problem about the subtraction of integers.

Ann's class is recording changes in temperature for science class. Students check the temperature readings each hour and determine the change in temperature. The chart shows the changes. What were the readings?

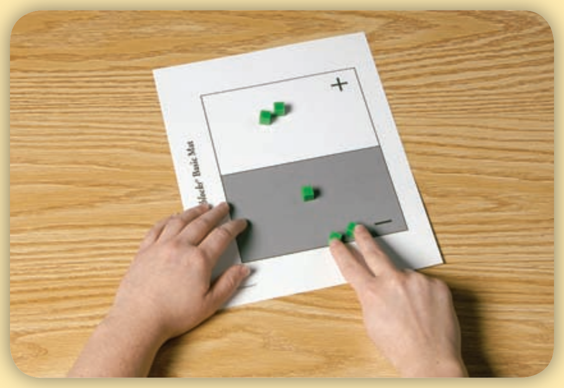
Time	8 A.M.	9 A.M.	10 A.M.	11 A.M.	Noon
Reading	+6°				
Change		Dropped 5°	Dropped 3°	Dropped 1°	Dropped 2°

Introduce the problem. Then have students do the activity to solve the problem. Write the chart on the board. Distribute the materials.

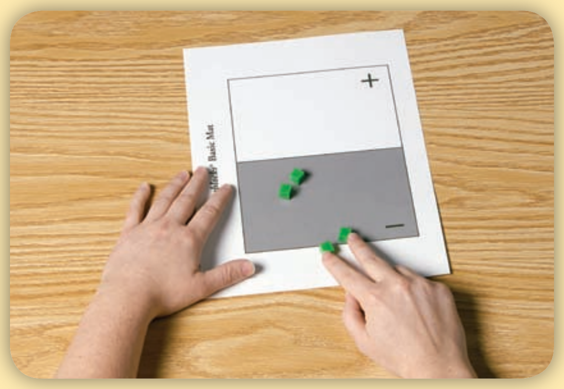
Materials
- Algeblocks® units
- BLM 4



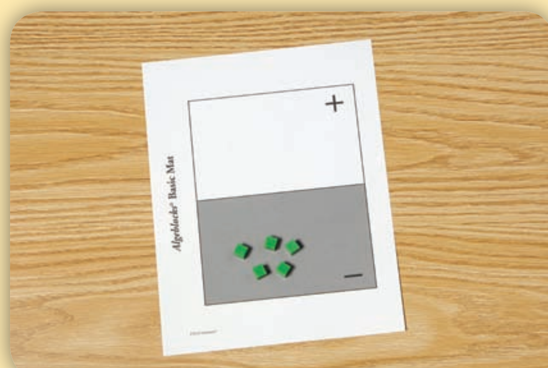
1. Have students place 6 unit blocks on the positive side of the mat. **Say:** *Read the first change in temperature, "Dropped 5°." Take away 5 cubes.* **Ask:** *What was the 9 A.M. reading?*



2. Have students read the next change in temperature, "Dropped 3°." **Ask:** *Since we don't have 3 blocks to remove, what should we do?* Instruct students to add zero pairs until they have 3 unit blocks in the positive section so they will have enough blocks to subtract from. **Say:** *You will need to add two zero pairs and then take away 3 blocks from the positive section, leaving 2 blocks on the negative section.* **Ask:** *What was the 10 A.M. reading?*



3. Have students read the next change in temperature, "Dropped 1°." **Ask:** *Since we don't have enough blocks to remove from the positive section (to subtract a positive 1), what should we do?* **Say:** *You will need to add one zero pair. Now take away 1 block from the positive side.* **Ask:** *What was the 11 A.M. reading?*



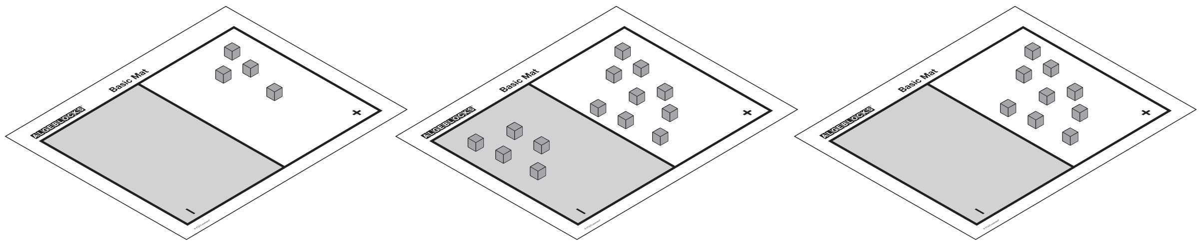
4. **Say:** *Now, represent the final temperature change of "Dropped 2°."* **Ask:** *What was the noon reading?*

Answer Key

Use Algeblocks unit blocks and a Basic Mat to model the integer subtraction sentence. Make zero pairs. Write the difference. Explain your work.

(Check students' work.)

1. 4 – (–5) = 9



Place 4 unit blocks on the positive side of the mat. Because there are not 5 unit blocks to remove from the negative side of the mat, add 5 zero pairs to the mat. Now take away 5 unit blocks from the negative side. That leaves 9 unit blocks on the positive side of the mat.

Using Algeblocks unit blocks and a Basic Mat, model each subtraction sentence. Sketch the model. Make zero pairs. Write the difference.

2. –6 – 4 = –10

3. –9 – 7 = –16

Find each difference.

4. –3 – (–1) = –2

5. 13 – (–7) = 20

6. 8 – (–12) = 20

7. –5 – 11 = –16

8. –1 – 6 = –7

9. 9 – (–8) = 17

10. 14 – (–16) = 30

11. –15 – (–15) = 0

Answer Key

Challenge! For the following subtraction problems, which ones require you to place additional unit blocks that equal zero pairs so that you can take away the number being subtracted? Explain.

$7 - 1$ $-7 - 1$ $7 - (-1)$ $-7 - (-1)$

Challenge: (Sample) $-7 - 1$ and $7 - (-1)$

LESSON 5

Objective

Multiply integers.

Common Core State Standards

- **7.NS.2a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

The Number System

Multiply Integers I

Students have developed the meaning of multiplication of whole numbers by using representations such as equal-size groups, arrays, area models, and equal jumps on a number line. Some of these representations also work for multiplication with negative numbers. Understanding multiplication of integers prepares students for division of integers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *When Ryan takes \$5 out of his savings account, the integer –5 is used to describe the change in the amount of money in the account.* **Ask:** *When Ryan donates \$5 to the food bank, what integer describes the change in the amount of money the food bank has?*
- **Say:** *The multiplication in this problem is 3 × (–5). Compare this with 3 × 5.* **Ask:** *How are they the same? How are they different?*

Solve It

Reread the problem with students. The amount of money in Ryan's savings account decreases each Friday, so a negative number (–5) is used to represent the change. To show the change in Ryan's account after 3 Fridays, students model the equation 3 × (–5) = –15. Have students explain the model.

More Ideas

For other ways to teach about multiplying integers—

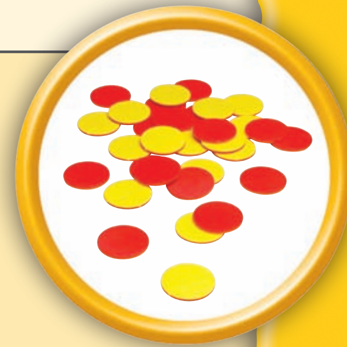
- Have students use yellow and red Centimeter Cubes to model this and similar problems.
- Summarize the rules for multiplying integers.
  (1) The product of two positive integers is positive.
  (2) The product of two negative integers is positive.
  (3) The product of a positive integer and a negative integer is negative.
- Using Two-Color Counters, have students model each rule. To model the product of two negative numbers, guide students to use repeated subtraction. To subtract groups of negative quantities from zero, first add red-yellow pairs. Then take away the red counters as appropriate.

Formative Assessment

Have students try the following problem.

In a computer game, you can win a maximum of 50 points and lose a maximum of 25 points in each round. What is the lowest possible score after three rounds?

A. –150 **B.** –75 **C.** –50 **D.** –25



Try It!

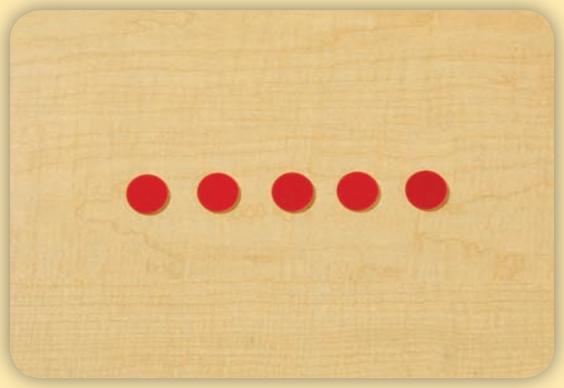
15 minutes | Pairs

Here is a problem about multiplying integers.

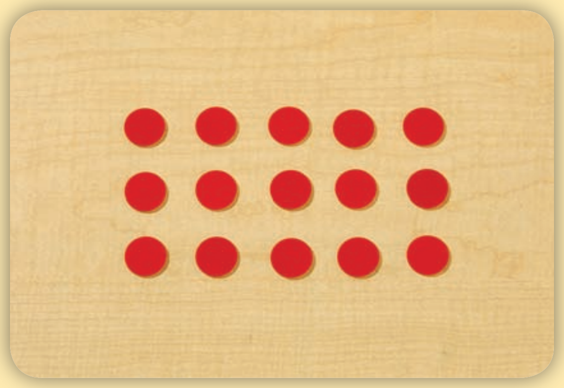
Ryan has a savings account. Every Friday he takes out $5 from the account and donates the money to the local food bank. What is the change in Ryan's account after three Fridays?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

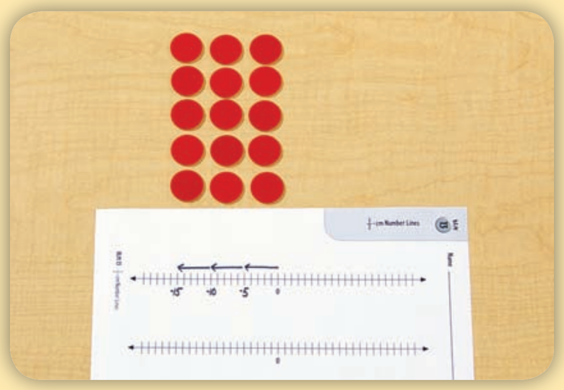
Materials
- Two-Color Counters (at least 20 per pair)
- BLM 5



1. Say: *Let each red counter represent one dollar donated—which is one dollar less in Ryan's savings account, or –1. Use counters to show the change in Ryan's account when he makes one donation.* Students display 5 red counters.



2. Say: *Now use counters to represent the change in Ryan's account after three Fridays. Organize the counters to show that there are three equal-size groups.* Students display 3 groups of red counters, with 5 in each group. **Ask:** *What amount of money is represented?*



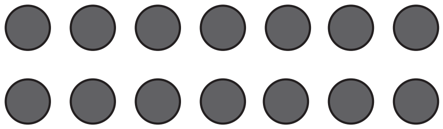
3. Say: *Now model this problem on a number line.* **Ask:** *How can you show that Ryan has $5 less in his savings account each Friday, for three Fridays?* Starting at 0, students jump 5 units left three times, ending at –15.

Look Out!

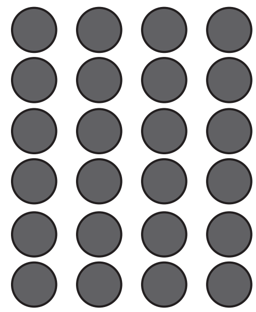
Sometimes students will be reluctant to think of multiplication as repeated addition when negative numbers are involved. Remind them that 3 × 5 is *3 groups of 5*, or 5 + 5 + 5, and that this idea applies to negative numbers, too. That is, 3 × (–5) is *three times negative five, or 3 groups of –5,* or (–5) + (–5) + (–5).

Answer Key

Use Two-Color Counters to model each multiplication problem. Use a number line to help. Write a number sentence for the product. (Check students' work.)

1. 

$2 \times (-7) = -14$

2. 

$6 \times (-4) = -24$

Using Two-Color Counters, model each multiplication problem. Sketch the model. Write the product.

3. $7 \times (-4)$ -28

4. $10 \times (-5)$ -50

Find each product.

5. $9 \times (-6)$ -54

6. $-5 \times (-7)$ 35

7. -4×11 -44

8. 9×7 63

9. $-3 \times (-1)$ 3

10. $8 \times (-7)$ -56

Answer Key

Challenge! What do you notice about the product when the factors have different signs? What do you notice about the product when the factors have the same signs? Draw pictures to help.

Challenge: (Sample) When both factors have the same sign, the product is positive. When the factors have different signs, the product is negative.

LESSON

6

Objective

Multiply integers.

Common Core State Standards

- **7.NS.2a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

The Number System

Multiply Integers II

Everyday situations that can be represented by negative numbers, such as money owed, points lost, and descent from an elevation, help students to gain facility in computing with integers. Once they can perform addition and subtraction of integers, students can advance to the multiplication of integers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What does* descent *mean? Would that be represented as a positive number or a negative number? How should we represent the descent of 4 meters for the first minute on the Algeblocks® Basic Mat?*
- **Ask:** *What number represents the location of the spelunker after she has descended for 2 minutes?*
- **Ask:** *You now have 6 groups of 4 unit blocks on the mat. What number represents the level, relative to the surface, of the spelunker after 6 minutes?*

Solve It

Reread the problem with the students. Discuss the meaning of *ascent* and *descent.* Make sure that students understand how to represent each direction with integers. Ask the students to write a math sentence to represent a descent of 4 meters per minute for 6 minutes: 6 × (–4) = –24.

More Ideas

For another way to teach about multiplication of integers—

- Have students draw a vertical number line with zero labeled at the top. Students should use Centimeter Cubes to mark the descent of the spelunker for each minute. Have them mark on the number line the position of the spelunker at the end of each minute. Ask them to repeat the activity for different numbers of minutes and different rates of descent.

Formative Assessment

Have students try the following problem.

A bank charges a penalty of $35 for each check returned for insufficient funds. Suppose that a customer miscalculates his balance and writes three bad checks. Which equation expresses the effect of the penalty on his checking account?

A. (–3) × (–35) = 105 **B.** 3 × (–35) = –105

C. 3 × 35 = 105 **D.** (–3) × 35 = –105

Try It! 15 minutes | Pairs

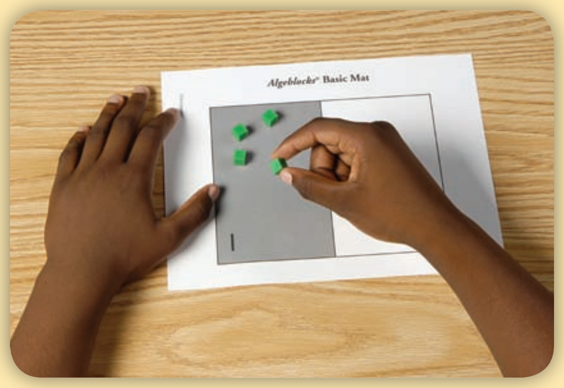
Here is a problem about multiplication of integers.

A spelunker descends into a crevasse at the rate of 4 meters per minute. What number represents her level, relative to the surface, after 6 minutes? Write a math sentence to describe this.

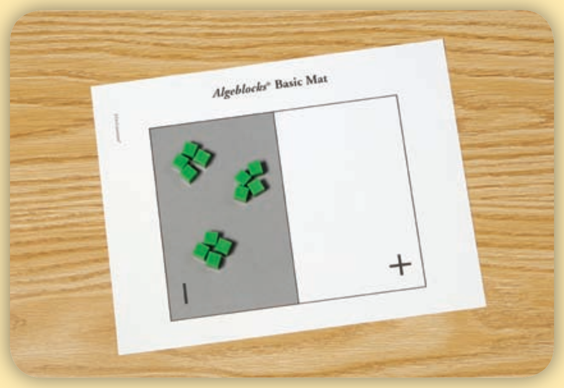
Introduce the problem. Then have students do the activity to solve the problem. Make sure that students understand how to represent a descent on their Algeblocks Basic Mats. Distribute the materials.

Materials

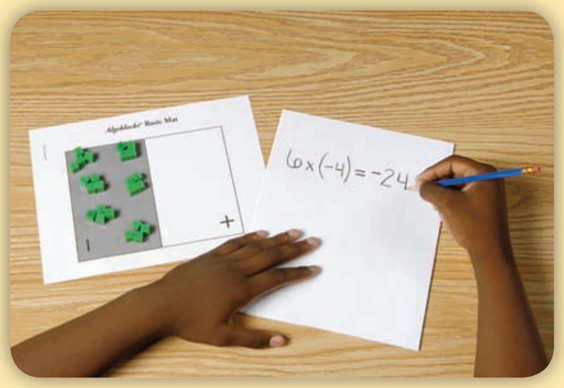
- Algeblocks® units
- BLM 4
- BLM 6
- Algeblocks Factor Track



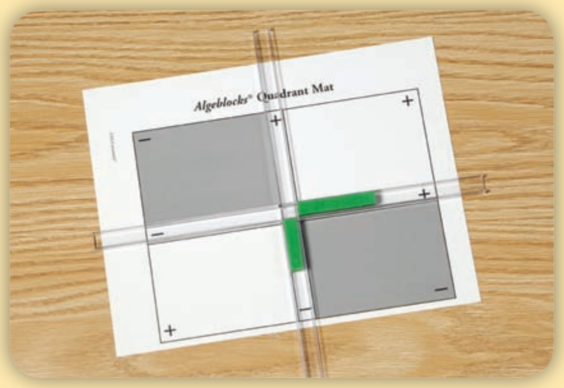
1. Have students place 4 unit blocks on the negative section of their Algeblocks Basic Mat. **Say:** *Now you have represented the location of the spelunker after the first minute of her descent.*



2. Say: *Continue placing groups of 4 unit blocks until you have represented the spelunker's 6-minute descent.*



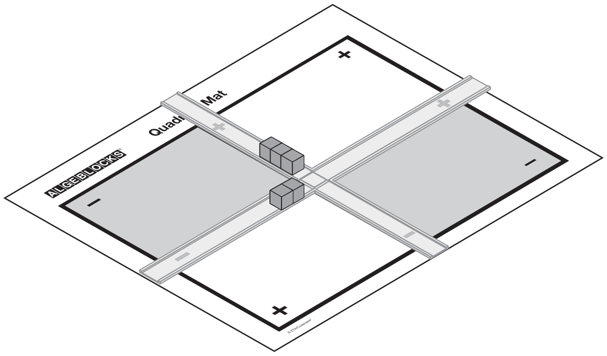
3. Have students complete putting 6 groups of 4 unit blocks on the left side of their mats. **Ask:** *How many unit blocks do you have on your mat? What is the depth of the spelunker after 6 minutes?* **Say:** *Write a math sentence to represent the spelunker's final location.*



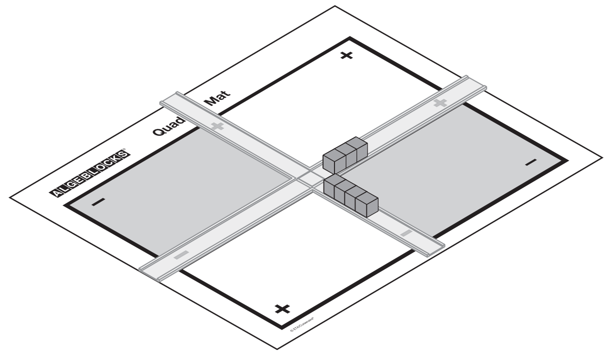
4. Have students use their Algeblocks Quadrant Mats and Factor Tracks. **Say:** *Represent 6 × (–4) on the Algeblocks Quadrant Mat.* Allow time for students to do this. **Say:** *Now remove the track and read your answer. Is it the same answer you got before?*

Use Algeblocks unit blocks, a Quadrant Mat, and a Factor Track. Model each integer multiplication sentence. Find each product. (Check students' work.)

1. –2 × 3 = –6

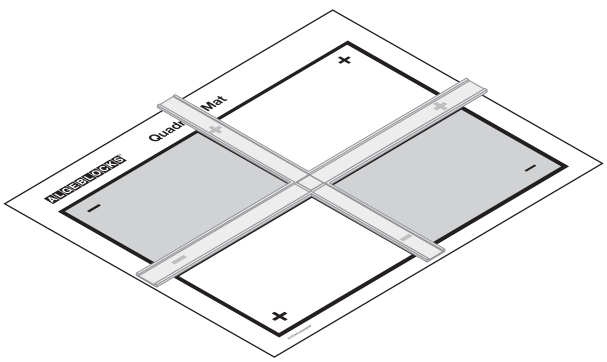


2. 3 × (–4) = –12

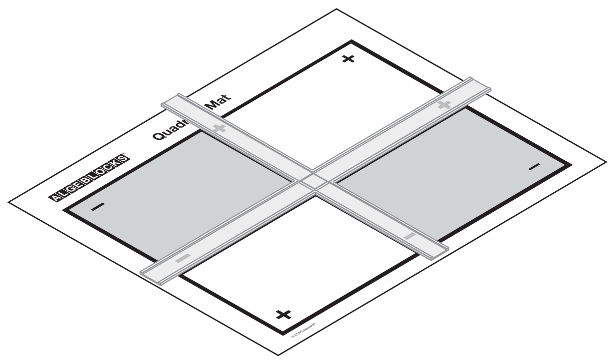


Using Algeblocks unit blocks, a Quadrant Mat, and a Factor Track, model each multiplication sentence. Sketch the model. Find the product.

3. –8 × (–2) = 16



4. 5 × (–4) = –20



Find each product.

5. 3 × (–6) = –18

6. –7 × (–3) = 21

7. –8 × 12 = –96

8. –9 × 5 = –45

9. –5 × (–6) = 30

10. 7 × (–8) = –56

11. 11 × (–6) = –66

12. –4 × (–1) = 4

Answer Key

Challenge! If the product of two integers is positive, what can you conclude about the factors? Draw a picture to help.

Challenge: (Sample) If the product is positive, that means that either both factors are positive or both factors are negative.

LESSON 7

Objective

Divide integers.

Common Core State Standards

- **7.NS.2b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

Divide Integers I

Students can use what they already know about multiplying integers to divide integers. Multiplication and division are inverse operations. So, for example, to find the quotient 30 ÷ 6, students can think of the related product: 6 × ? = 30. The rules for division of positive and negative numbers are the same as those for multiplication.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *The addition problem for finding the sum of the scores is –4 + 1 + (–3).* **Ask:** *Does it matter which two numbers you add first?*
- **Say:** *The division problem for finding the average score is –6 ÷ 3 = –2. Write a related multiplication problem.* Students can write either *3 × (–2) = –6*, or *(–2) × 3 = –6.*
- **Ask:** *Looking at the three scores, is it reasonable that the answer is negative rather than positive?* Have students explain their answers.

Solve It

Reread the problem with students. The average score is the sum of the scores divided by the number of scores: [–4 + 1 + (–3)] ÷ 3. The sum of the scores is –6 and –6 ÷ 3 is –2. Ken's average score is –2.

More Ideas

For other ways to teach about dividing integers—

- Have students use Centimeter Cubes to model pairs of number sentences, such as –10 ÷ 5 = –2 and 5 × (–2) = –10. Suggest that students use red cubes for negative numbers and yellow cubes for positive numbers. Also, discuss why –8 ÷ 4 is easier to model than 8 ÷ (–4).
- Summarize the rules for dividing integers, and note that they are the same as the rules for multiplying integers.

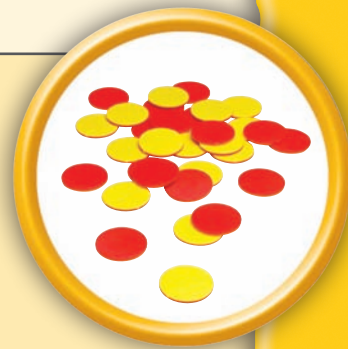
  (1) The quotient of two positive integers is positive.
  (2) The quotient of two negative integers is positive.
  (3) The quotient of a positive integer and a negative integer is negative.
- Using Two-Color Counters, have students model an equation for rules 1 and 3.

Formative Assessment

Have students try the following problem.

Find the average of –6, –4, 8, and –2.

A. –4　　**B.** –1　　**C.** 1　　**D.** 4



Try It! 15 minutes | Pairs

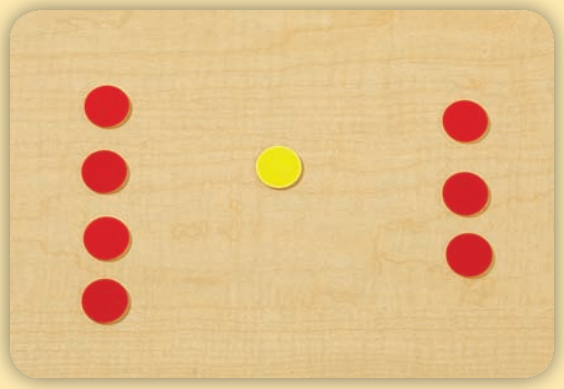
Here is a problem about dividing integers.

In three rounds of golf, Ken shot scores of –4, +1, and –3. What was his average score?

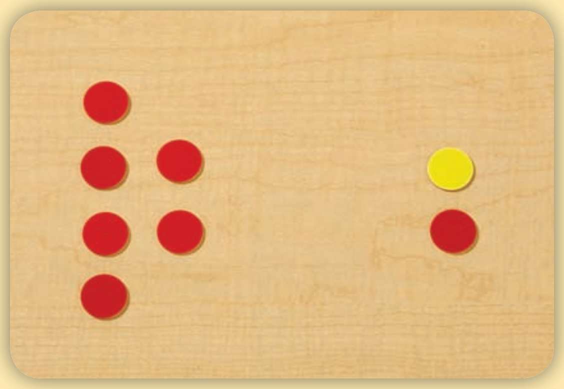
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials. Remind students that the average is the sum of the scores divided by the number of scores.

Materials

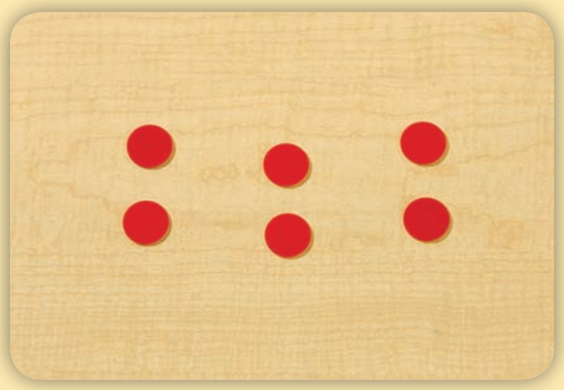
- Two-Color Counters (at least 20 per pair)



1. Say: *Each red counter represents negative one, and each yellow counter represents positive one. Use counters to show Ken's three scores.* Students place 4 red counters together, 1 yellow counter by itself, and 3 red counters together.



2. Say: *To find the average score, you first need to add the three scores. Use the counters to find the sum.* Students pair one yellow counter with a red counter to equal zero, and move the pair aside. There are 6 red counters left. The sum is –6.



3. Say: *Now divide the sum by 3, since there are 3 scores. Divide the counters that represent the sum into 3 equal groups.* **Ask:** *How many counters are in each group?* Students arrange the 6 red counters into 3 groups, with 2 red counters in each group.

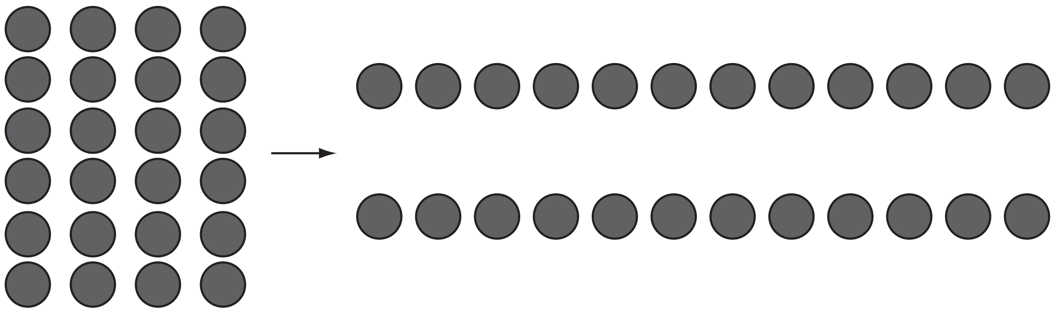
⚠ Look Out!

Make sure students understand that the average does not have to be one of the scores. To calculate the average score, they must add all the scores and divide by the number of scores, even though the scores include both positive and negative numbers. Note that the given scores, written in order from least to greatest, are –4, –3, and +1. It makes sense that the average, which is –2, lies between the least score, –4, and the greatest score, +1.

Answer Key

Use Two-Color Counters to model each division problem. Write a number sentence for the quotient.

(Check students' work.)

1. 

$-24 \div 2 = -12$

Using Two-Color Counters, model each division problem. Sketch the model. Find the quotient.

2. $-35 \div 7$

-5

3. $-81 \div 9$

-9

Find each quotient.

4. $49 \div (-7) = -7$

5. $-45 \div (-5) = 9$

6. $-42 \div 7 = -6$

7. $9 \div (-3) = -3$

8. $-30 \div (-6) = 5$

9. $28 \div (-7) = -4$

Answer Key

Challenge! How do the rules for dividing integers differ from the rules for multiplying integers? Draw pictures to help.

Challenge: (Sample) The rules for multiplying and dividing integers are the same.

LESSON 8

Divide Integers II

Objective

Divide integers.

Common Core State Standards

- **7.NS.2b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

To extend their understanding of integers to division of integers, students need to draw on their understanding of multiplication and division as inverse operations. Using the area-array model for multiplication will help students use the Algeblocks® Quadrant Mat effectively.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What integer would represent a $14 loss? Where should we place the 14 unit blocks?*
- **Ask:** *What are the factors of 14?*
- **Ask:** *Where should we place the divisor of 2?*
- **Ask:** *What would the other factor, the quotient, be?*

Solve It

Reread the problem with the students. Help them represent the financial loss on their Algeblocks Quadrant Mats. Suggest that they make groups of two unit blocks, since there are two business partners. The number of groups represents the number of dollars each partner lost. Have students write a math sentence to represent the situation.

More Ideas

For another way to teach about division of integers—

- Challenge students to write a number sentence and represent it using the Algeblocks Quadrant Mat and Factor Track for each of the following situations:

  a. a positive number divided by a positive number;

  b. a positive number divided by a negative number;

  c. a negative number divided by a positive number;

  d. a negative number divided by a negative number.

Formative Assessment

Have students try the following problem.

The water level in a swollen river falls 30 cm in 5 hours. What is the average change in its level each hour?

A. 15 cm per hour **B.** 6 cm per hour

C. –15 cm per hour **D.** –6 cm per hour

Try It! 15 minutes | Pairs

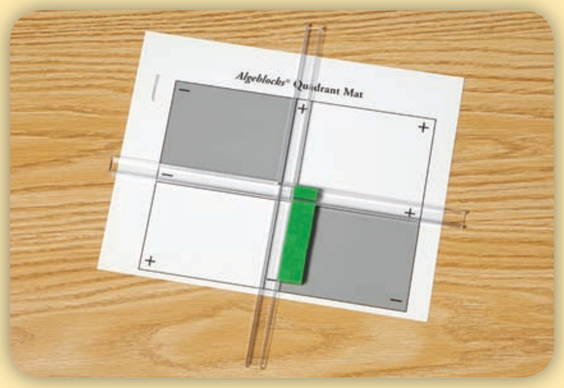
Here is a problem about dividing a negative integer.

Jessica and Taylor decided to start a dog-walking business for the summer. After paying for lessons in dog handling and some basic equipment, they began signing up customers. At the end of the summer, they added up the money they had earned, subtracted their expenses, and found that they had actually lost \$14! What is the financial outcome of this business venture for each of the two partners?

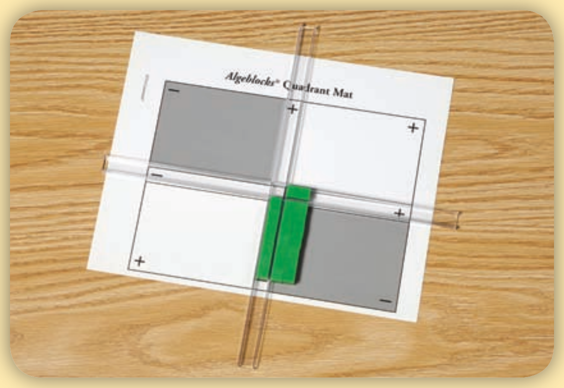
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

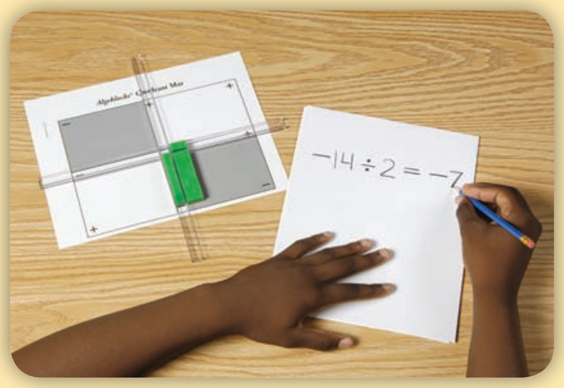
- Algeblocks® units
- BLM 6
- Algeblocks Factor Track



1. Have students represent on their Algeblocks Quadrant Mat the money lost in the business venture. **Ask:** *What are the factors of 14?* **Ask:** *How many business partners were there?* Have students place 2 unit blocks on the positive side of the horizontal bar of the Factor Track. **Say:** *Now, arrange 14 unit blocks in Quadrant IV in a rectangle with 2 unit blocks on one side.*



2. Ask: *How many groups of 2 do you have?* Have students place 7 unit blocks on the negative part of the vertical bar of the Factor Track. **Say:** *Now you have shown that each partner lost \$7.*



3. Have students write a number sentence to represent each partner's loss.

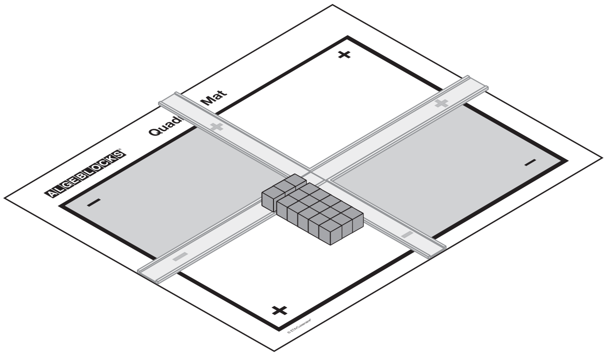
⚠ Look Out!

Watch for students who are unsure of where to place the 14 unit blocks on the Quadrant Mat. Since the business partners lost \$14, the 14 blocks should go in either of the gray areas. (If students choose to put the blocks in Quadrant II, they should take care to place the two blocks, representing two partners, on the positive area of the Factor Track.) In this example, the 14 blocks were placed in the lower right and the other blocks were placed accordingly.

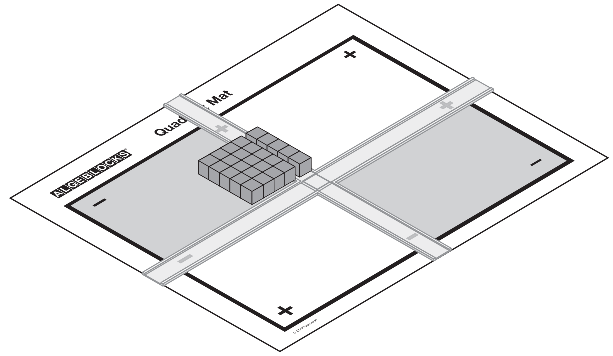
Answer Key

Use Algeblocks unit blocks, a Quadrant Mat, and a Factor Track. Model each integer division sentence. Find each quotient. (Check students' work.)

1. 15 ÷ (–3) = –5

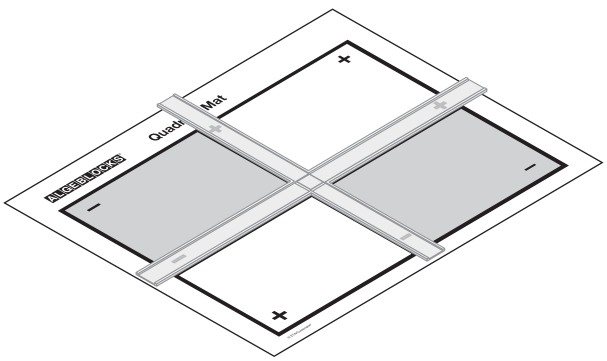


2. –20 ÷ (5) = –4

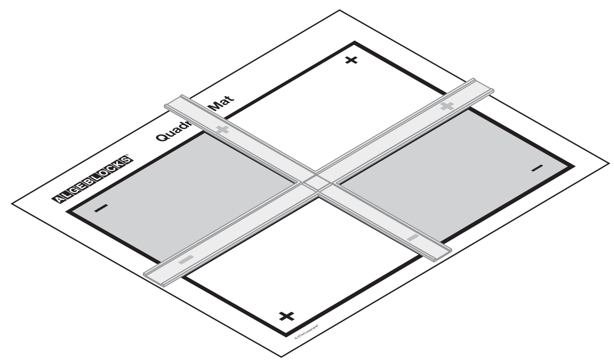


Using Algeblocks unit blocks, a Quadrant Mat, and a Factor Track, model each division sentence. Sketch the model. Find each quotient.

3. –28 ÷ (–7) = 4



4. 45 ÷ (5) = 9



Find each quotient.

5. 36 ÷ (–6) = –6

6. –27 ÷ (–3) = 9

7. –18 ÷ 3 = –6

8. –49 ÷ 7 = –7

9. –35 ÷ 5 = –7

10. 12 ÷ (–2) = –6

11. –24 ÷ (–4) = 6

12. –5 ÷ (–5) = 1

Answer Key

Challenge! How do the rules for adding and subtracting integers differ from the rules for multiplying and dividing integers?

Challenge: (Sample) When adding or subtracting, the sum or difference depends on the magnitude of the numbers. A positive plus a negative can be either positive or negative. When mutliplying or dividing, the product or quotient of a negative and a positve will always be negative.