Geometry

The focus for students in geometry at this level is reasoning about area, surface area, and volume. Students also learn to work with visual tools for representing shapes, such as graphs in the coordinate plane and nets.

Area is a measure of the amount of surface enclosed by a bounding perimeter. The defining unit of area is the *unit square*, which can be a square inch or a square of any other standard unit or arbitrary unit. Students begin their study of area by learning about the notion of the unit square and applying it to find the areas of squares and other shapes that can be decomposed into squares. At this level, students find the areas of triangles, special quadrilaterals, and other polygons.

The word area, by itself, usually means the area of a polygon, circle, or other distinct, planar surface. The area of the surface of a three-dimensional object, or solid, is called ***surface area***. If the solid is a polyhedron, for example, then the surface area is the sum of the areas of all its faces. A "flattened out" representation of the solid, called a *net*, can be used to visualize its surface area. At this level, students make nets for and find surface areas of prisms and pyramids.

Volume is the space filled, or occupied, by a three-dimensional object. The basic unit of volume is the *unit cube*. Students can study volume by using unit cubes to mentally or physically build up the objects they want to investigate. This works best with rectangular prisms, but the insight gained can be applied to more general cases.

The Grade 6 Common Core State Standards for Geometry specify that students should–

- Solve real-world and mathematical problems involving area, surface area, and volume.

The following hands-on activities will help students internalize concepts in geometry. They will help students see, for example, that area is a property of surfaces, no matter how the surfaces are situated. This is an example of finding structure in mathematics. Students in geometry at this level will work with many formulas. Hands-on activities help students see why the formulas work.

Geometry

Contents

LESSON 1

Objective

Compare the areas of parallelograms and rectangles.

Common Core State Standards

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Geometry

Area of a Parallelogram

In this activity, students reason about the area of a parallelogram by visualizing its relationship to a corresponding rectangle. Without a formula, the area of a rectangle can be found by counting the number of square units that fill it. But since a parallelogram does not have right angles, it cannot be filled with whole squares. An area formula is needed. The area formula for parallelograms is a general form of the formula for rectangles.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How did the shapes of the parking spaces change on the geoboard when you shifted the bottoms to the right?*
- **Ask:** *How can you reason that the new shapes have the same area as the original shapes?*
- Explain how the area formula remains essentially the same except that the length of the rectangle is replaced by the height of the parallelogram.

Solve It

Reread the problem with students. Have students write a paragraph that describes how the area formula for a parallelogram is a general form of the area formula for a rectangle.

More Ideas

For other ways to teach about areas of parallelograms—

- Using the square and two small triangle Tangrams, demonstrate how a rectangle can be made into a parallelogram having the same height and base length, and discuss why the areas are the same.
- Students can use 4 AngLegs® pieces to form a rectangle, then push on opposite corners to slant the rectangle and form a parallelogram. Ask students to compare the area of the rectangle with the area of the parallelogram. The area of the parallelogram is less because the height has decreased.

Formative Assessment

Have students try the following problem.

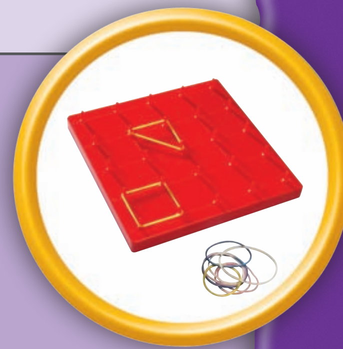
A parallelogram has an area of 20 square inches and a base length of 4 inches. What is the height?

A. 4 inches

B. 5 inches

C. 6 inches

D. 10 inches



Try It! 20 Minutes | Pairs

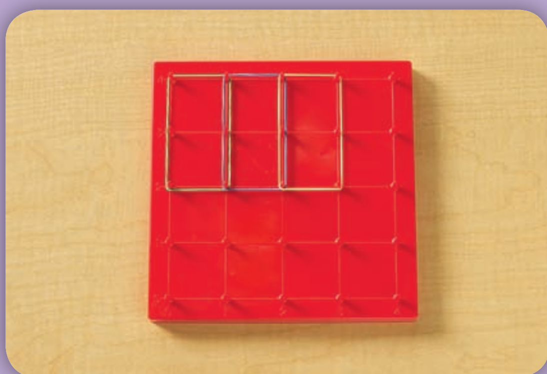
Here is a problem about areas of parallelograms.

Doug wants to install three parking spaces on part of a rectangular section of land in front of his store. The land is 18 feet deep by 36 feet wide. Doug considers installing perpendicular spaces that are 9 feet wide. He also considers angled spaces. Does Doug's choice of layout affect the amount of paving that he needs for the parking spaces?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Geoboards, rubber bands, grid paper, and colored pencils to students.

Materials

- Geoboards (1 per pair)
- rubber bands (6 per pair)
- 10 x 10 Grid (BLM 9; 1 per pair)
- colored pencils (2 colors per pair)



1. Say: *Using the upper half of a geoboard to represent the section of land, model three perpendicular parking spaces.* If necessary, guide students to realize that each space should be depicted using a 2-by-1 rectangle.

2. Say: *Draw a 2-by-4 rectangle on a grid to represent the section of land. Draw and shade three rectangles to represent the three perpendicular parking spaces.* **Ask:** *What is the area of each rectangle? What is the area of the three rectangles added together?* Have students write their answers.

3. Say: *On the geoboard, shift the bottom of each rectangle to the right by one unit to create a model of three angled parking spaces.* **Ask:** *What is the area of each parallelogram and all three parallelograms together?* Have students draw the model on their grid. Guide them to see that a triangular area has been uncovered on the left but that an equal area has been added on the right.

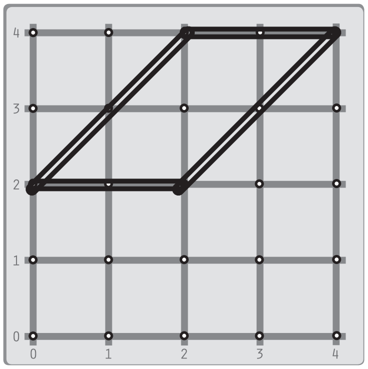
⚠ Look Out!

Point out to students that only one side length (called the base) is used to find the area of a parallelogram. The other measurement used is the height of the parallelogram, which is not a side length. Note that any of the four sides of a parallelogram can be called a base and the base is perpendicular to the height. The base and height are multiplied to find the area. With a rectangle, both the base and height (or length and width) happen to be sides of the rectangle.

Answer Key

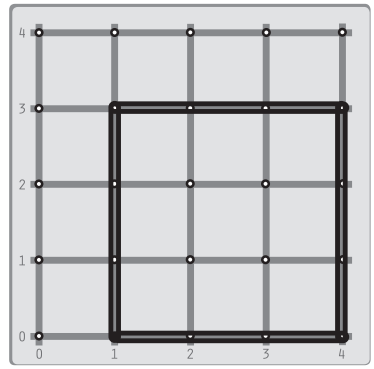
Use a Geoboard to model each parallelogram. Find its area. (Check students' work.)

1.



4 square units

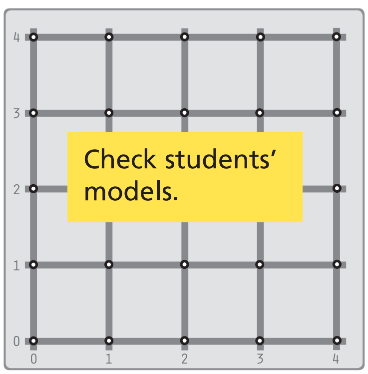
2.



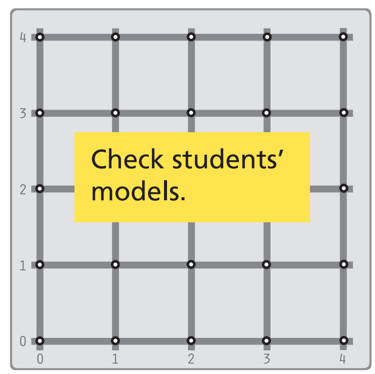
9 square units

Using a Geoboard, model a parallelogram with the given area. Sketch the shape.

3. 16 units^2

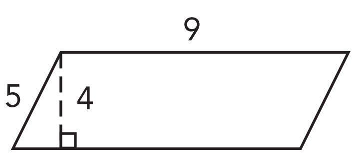


4. 12 units^2



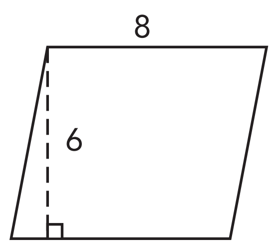
Find the area of each parallelogram.

5.



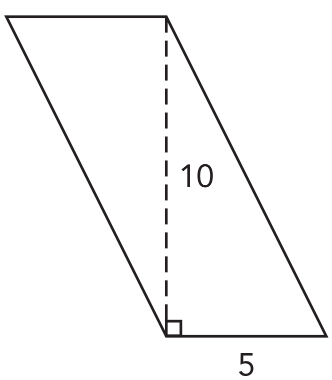
36 units^2

6.



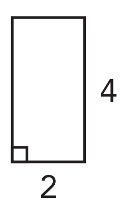
48 units^2

7.



50 units^2

8.



8 units^2

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Answer Key

Challenge! How is finding the area of a parallelogram different from finding the area of a rectangle? How is it similar? Draw a picture to help.

Challenge: (Sample) The height of a parallelogram may or may not be one of the sides. The height of a rectangle is always one of the sides. To find the area of both, you multiply the base by the height. In a rectangle, the base and height are named the length and width.

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LESSON 2

Objective

Observe and generalize about the area enclosed within a constant perimeter.

Common Core State Standards

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Geometry

Constant Perimeter and Changing Area

The concepts of perimeter and area are often misunderstood (and sometimes confused) by students who tend to lack real-world experience, such as tiling floors, painting walls, and fencing gardens. This activity gives students hands-on experience with the concept of constant perimeter and addresses the effect that a change in dimension has on the enclosed area.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What was the area of the first "garden" you made with the fencing? What was its perimeter?*
- **Ask:** *What were the areas of the other gardens you made? What were the perimeters?*
- **Ask:** *What conclusion(s) can you draw about perimeter and maximum area (of a rhombus) from this lesson?* (The square has the maximum area.)

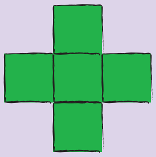
Solve It

Reread the problem with students. Ask them to explain in writing the maximum area possible for a rhombus with a given perimeter. Have them include sketches.

More Ideas

For other ways to teach the concept of constant perimeter and maximum area—

Length	Width	Area
5	1	
4	2	
3	3	



- Have students use Color Tiles to create rectangles with a perimeter of 12 units. Have them use a T-chart to record the areas of the rectangles. Then have students create other shapes with a perimeter of 12 units (see example). What is the area of each?
- Have students use the XY Coordinate Pegboard to create a rectangle with a perimeter of 16 units. Then have them create different rectangles with the same perimeter until they have made all of the possible rectangles. Which rectangle has the greatest area? Which has the smallest area?

Formative Assessment

Have students try the following problem.

What is the largest rectangular area that can be enclosed with 24 feet of fencing?

A. 11 sq. ft **B.** 14 sq. ft **C.** 24 sq. ft **D.** 36 sq. ft

Try It! 20 minutes | Pairs

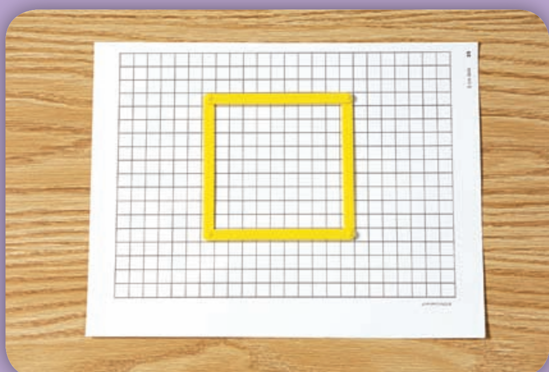
Here is a problem about finding the maximum area that can be contained by a quadrilateral with a given perimeter.

Jesse and Keshia have four 10-foot pieces of edging. They want to use the uncut pieces to enclose a vegetable garden. Keshia wants the garden to be a rhombus so they can avoid the birdbath. Jesse wants a square garden so that it will enclose as large an area as possible. Keshia insists that it doesn't matter. As long as the perimeter is still 40 feet, the area it encloses will always be the same. Who is right?

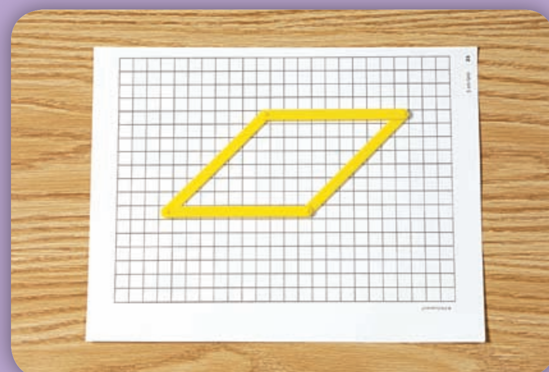
Introduce the problem. Then have students do the activity to solve the problem. Distribute AngLegs and grid paper to students.

Materials

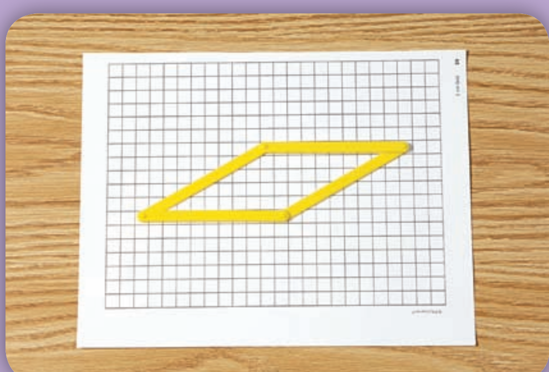
- AngLegs® (4 yellow)
- Centimeter Grid Paper (BLM 10; 1 per pair)



1. Have students construct a rectangle using 4 yellow (10 cm) AngLegs. Have the students place the figure on the grid paper. The figure will be a square, 10 cm on a side. Have students use the formula for the area of a parallelogram ($A = bh$) to find its area.



2. Now have students "collapse" the parallelogram to a height of 8 cm without moving the base. They should use the formula to find the area of the new parallelogram.



3. Have students continue reducing the height of the parallelogram to 6 cm and then to 4 cm. Each time, they should calculate the area of the new parallelogram they form.

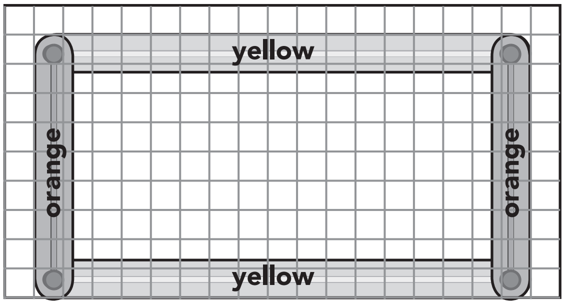
⚠ Look Out!

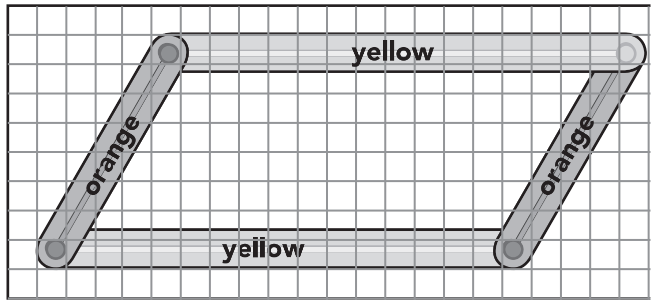
Students may need to review the definitions of *parallelogram* and *rhombus*. They also may need to review the formula for finding the area of a parallelogram. Also, explain to students that the AngLegs have a groove running lengthwise down the midpoint of each leg. Encourage them to place the AngLegs on the grid paper so that the groove coincides with the lines of the grid whenever possible. This will make it easier for them to determine the dimensions of the various parallelograms.

Answer Key

Use AngLegs® and grid paper to model the shapes shown. Find the perimeter of each shape. Find the area of each shape. (Check students' work.)

1.





Perimeter of rectangle __52__ units

Area of rectangle __153__ sq units

Perimeter of parallelogram __52__ units

Area of parallelogram __136__ sq units

Using AngLegs and grid paper, model two shapes that have the given perimeter, but different areas. Name the area of each shape.

2. 50 units

Figure 1

Figure 2

Check students' models; answers will vary depending on models.

Area of Figure 1 ____________

Area of Figure 2 ____________

Find the perimeter and area of each figure.

3. Figure 1 Figure 2 Figure 3

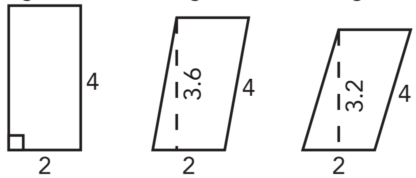


Figure 1: 12 units; 8 units²;
Figure 2: 12 units; 7.2 units²;
Figure 3: 12 units; 6.4 units²

4. Figure 1 Figure 2 Figure 3

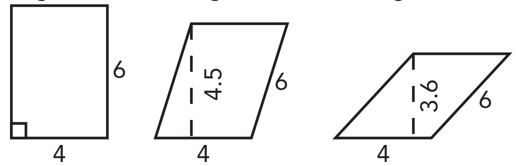


Figure 1: 20 units; 24 units²;
Figure 2: 20 units; 18 units²;
Figure 3: 20 units; 14.4 units²

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Answer Key

Challenge! How can a rectangle with side lengths of 6 and 10 have a different area than a parallelogram with side lengths of 6 and 10? What do you know about their perimeters? Draw a picture to help.

Challenge: (Sample) In the rectangle, the height is equal to one of the side's lengths. In the parallelogram, the height is not equal to one of the side's lengths. So, when you multiply length times height for the rectangle, you are multiplying different numbers than you do when you multiply the length times the height for the parallelogram. Both shapes have the same perimeters because the lengths of their sides are congruent.

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LESSON 3

Objective

Compare the areas of triangles and parallelograms.

Common Core State Standards

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Geometry

Area of a Triangle

The area of a parallelogram is $b \times h$, or *base times height.* Squares, rectangles, and parallelograms can all be divided in half to form congruent triangles. Any triangle made in this way has an area that is half the area of the original figure. Most students already have experience with this fact. For example, they might have seen a square sandwich cut in half to make two triangles. They can probably reason that each triangle has an area that is half the area of the whole sandwich. The same reasoning can be applied using a parallelogram as the original figure. This leads to the general formula for the area of a triangle: $A = \frac{1}{2} \times b \times h$.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How can you compare the area of the triangle with the area of the parallelogram without using formulas?*
- **Ask:** *If you know that the area of a parallelogram is* b × h*, how do you reason that the area of a triangle is* $\frac{1}{2} \times \mathrm{b} \times \mathrm{h}$*?*
- **Ask:** *When is the height of a triangle equal to the length of a side?*

Solve It

Reread the problem with students. The area of the triangle is half the area of the related parallelogram. So the area of the triangle is 1 square unit and the area of the parallelogram is 2 square units.

More Ideas

For other ways to teach about the area of a triangle—

- Have students use Pattern Blocks to see the relationship between the area of a triangle and the area of a parallelogram. Two green triangles placed together cover the same area as one blue parallelogram.
- Have students use a Geoboard to design a quilt block with various shapes of squares, rectangles, triangles, and parallelograms. Have them complete a table for the quilt block with the columns *Color, Shape, Area,* and *Total Area in Quilt Block*. Students can copy their quilt blocks onto grid paper and color them, then exchange blocks and analyze the areas of particular colors.

Formative Assessment

Have students try the following problem.

A parallelogram has a base of 7 feet and a height of 4 feet. What is the area of a triangle formed by drawing a diagonal on the parallelogram?

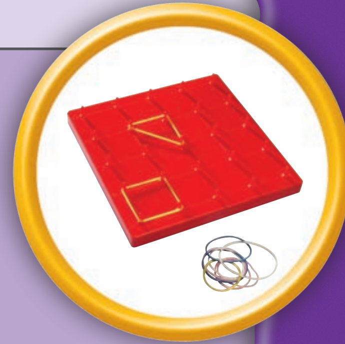
A. 7 square feet

B. 14 square feet

C. 28 square feet

D. 56 square feet

Try It! 30 Minutes | Groups of 4



Here is a problem about the area of a triangle.

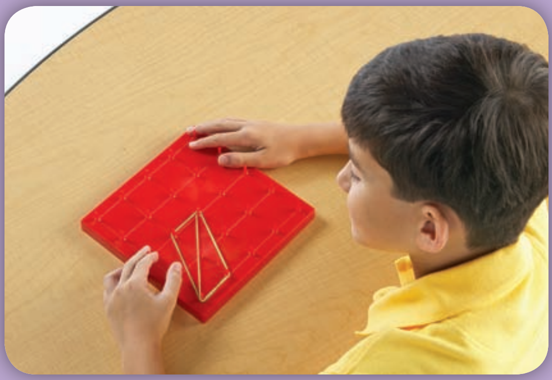
Samira has made a triangle on a geoboard. Jo says she can find the area of the triangle by changing it into a parallelogram. How might Jo do this?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Geoboards, rubber bands, grid paper, paper, and pencils to students.

Materials

- Geoboards (1 per group)
- rubber bands (2 per group)
- Centimeter Grid Paper (BLM 10; 1 sheet per group)
- paper (1 sheet per group)
- pencils (1 per group)

1. Say: *Make a triangle on the geoboard using the points (0, 0), (1, 0), and (2, 2).* Students use a rubber band to make the triangle. Have students place a second rubber band over the first.



2. Say: *Now create a parallelogram with one of the rubber bands. Stretch the rubber band to create a fourth corner at the point (1, 2).* One of the rubber bands is stretched to the point (1, 2). **Say:** *Compare the triangle with the parallelogram.* Elicit that the triangle is half of the parallelogram.

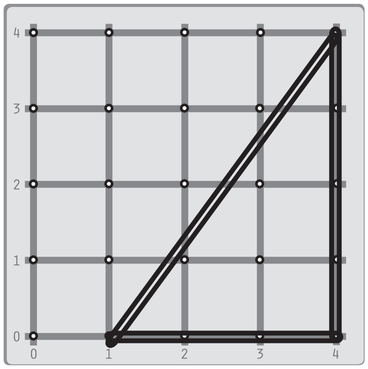
3. Ask: *What is the formula for finding the area of a parallelogram?* Elicit that the formula is $A(▱) = b \times h$. **Ask:** *What is the formula for finding the area of a triangle?* **Say:** *Use the models on the geoboard to help you determine the formula.* Elicit that the formula is $A(\triangle) = \frac{1}{2} \times b \times h$.

4. Say: *Draw the triangle and parallelogram on grid paper and compare their areas.* **Ask:** *What is the area of each shape?*

Answer Key

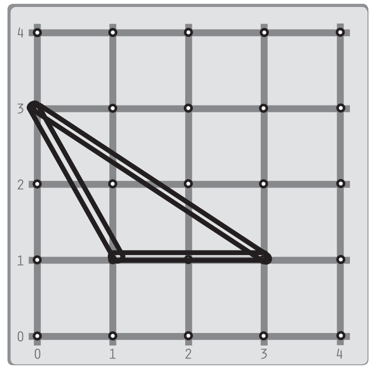
Use a Geoboard to model each triangle. Find its area. (Check students' work.)

1.



6 square units

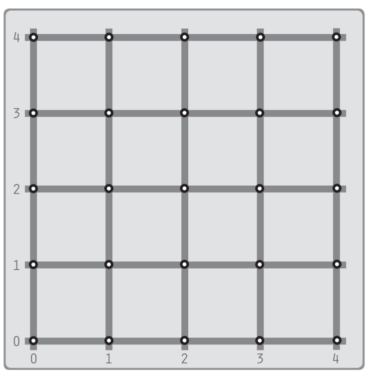
2.



2 square units

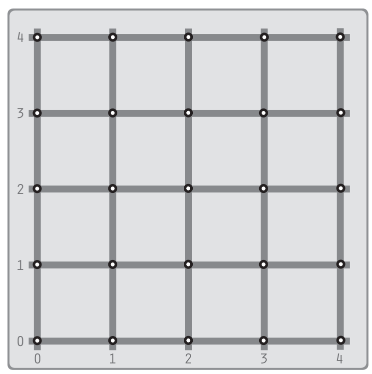
Using a Geoboard, model each triangle. Sketch the model. Find its area.

3. base: 4 units, height: 2 units



4 square units

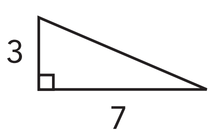
4. base: 4 units, height: 4 units



8 square units

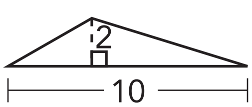
Find the area of each triangle.

5.



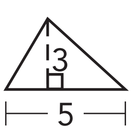
10.5

6.



10

7.



7.5

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Answer Key

Challenge! Explain why the formula for the area of a triangle includes the fraction $\frac{1}{2}$. Draw a picture.

Challenge: (Sample) You can duplicate a triangle and rotate it 180° to form a rectangle that has a length equal to the base of the triangle and a width equal to the height of the triangle. The size of the original triangle is only half the size of the rectangle. So the area of the triangle is half of the area of the rectangle.

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LESSON 4

Objective

Determine the area of a trapezoid by dividing it into two triangles whose heights are the same as the height of the trapezoid and whose bases are those of the trapezoid.

Common Core State Standards

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Geometry

Area of Trapezoids

By the time students reach 7th and 8th grade, they are expected to bring to their study of geometry and measurement an understanding of common figures such as squares, rectangles, and triangles. Also, students at this level are able to analyze new shapes by comparing them to shapes they are already familiar with and by combining previously learned processes. The study of trapezoids affords students the opportunity to build on what they know about triangles and to make the transition from these common figures to more irregular ones.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Have students look at the trapezoid and triangles they made on the XY Coordinate Pegboard and drew on the centimeter dot paper. **Ask:** *In what ways are the trapezoid and its internal triangles similar?*
- **Ask:** *If you divide the trapezoid into two different triangles by connecting points (4, 9), (9, 9), and (13, 5), will your answer to the problem be the same? Explain.*
- **Ask:** *What other shapes could you divide the trapezoid into in order to find its area?*

Solve It

Reread the problem with students. Have students write a sentence that states the key to finding the area of a trapezoid. When they have written and discussed this with their classmates, have them propose a formula for finding the area of any trapezoid $\left(A = \frac{1}{2}h\,(b_1 + b_2)\text{, where } b_1 \text{ and } b_2 \text{ are the bases}\right)$.

More Ideas

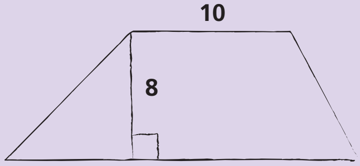
For other ways to teach about the area of trapezoids—

- Have students work in pairs to construct trapezoids from Pattern Blocks. They should then transfer the figures to $\frac{1}{8}$" dot paper. Have students determine the area of the trapezoid(s) they make. (Note: The sides of all of the Pattern Blocks are 1 or 2 inches long.)
- Have students work in pairs to construct trapezoids from Tangram pieces. Using the square as a unit, have students determine the area of the trapezoid(s) they make. (Note: 2 small triangles = 1 square.) Have students make a right trapezoid from some of the pieces.

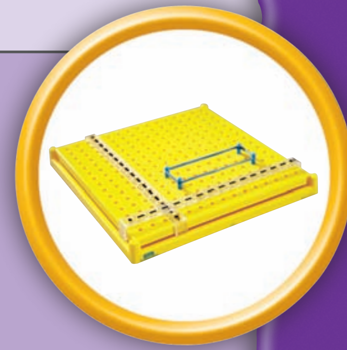
Formative Assessment

Have students try the following problem.

Find the area of the trapezoid shown here.



A. 52 sq. units **B.** 128 sq. units **C.** 176 sq. units **D.** 256 sq. units



Try It! 35 minutes | Pairs

Here is a problem about finding the area of a trapezoid.

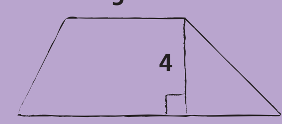
Steven works for a landscaping company that will be installing sod in a customer's backyard. The area to be covered is a trapezoid. One base of the trapezoid is 11 yards long, and the other is 5 yards long. The trapezoid is 4 yards high. How many square yards of sod will Steven need to cover the area?

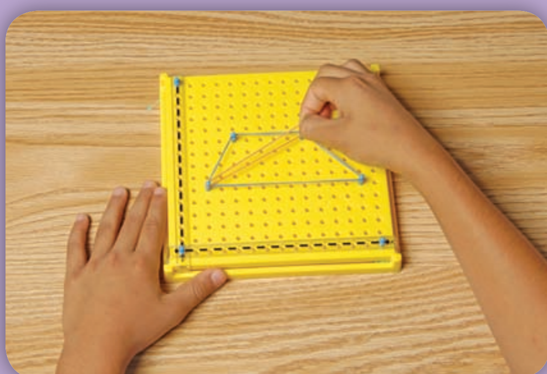
Introduce the problem. Then have students do the activity to solve the problem. Review with students the features of a trapezoid. Sketch the trapezoid on the board as shown.

Materials

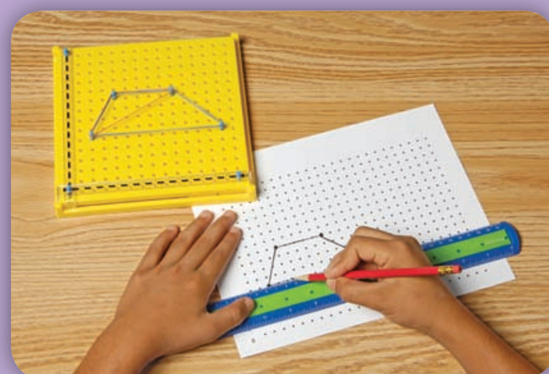
- XY Coordinate Pegboard
- Centimeter Dot Paper (BLM 11; 1 per pair)
- ruler or straightedge

Distribute the materials. Have students slide the *x*-axis of the XY Coordinate Pegboard to the bottom of the board and the *y*-axis to the left edge and secure the axes with pegs. Write the following coordinates on the board: (2, 5), (4, 9), (9, 9), (13, 5). Have students place pegs at the coordinates and stretch a rubber band around the pegs to create a trapezoid.





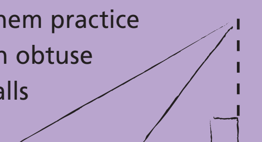
1. Have students divide the trapezoid into two triangles by stretching a rubber band of a different color from (2, 5) to (4, 9) to (9, 9).



2. Have students draw the trapezoid on centimeter dot paper. Then have them draw a line that divides the trapezoid into two triangles.

Look Out!

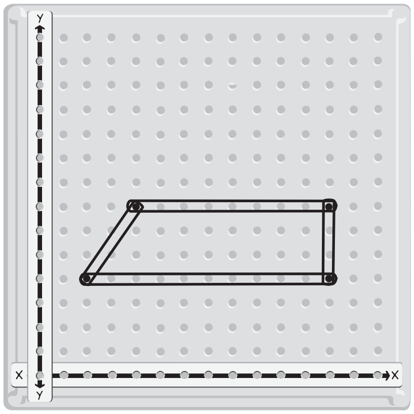
Students may have difficulty recognizing that the height of each of the trapezoid's two internal triangles is the same as the height of the trapezoid itself. Have them practice identifying the height of an obtuse triangle when the height falls outside the figure.



3. Students should determine the length of the top base of the trapezoid, the bottom base, and the altitude by counting the units. Students should then find the area (in square units) of each of its internal triangles ($A = \frac{1}{2}bh$). Note that the bases of the triangles are also the bases of the trapezoid. Students can add the areas of the triangles together to determine the area of the trapezoid.

Use an XY Coordinate Pegboard to model the trapezoids. Divide each trapezoid into two triangles. Find the area of each trapezoid. (Check students' work.)

1.

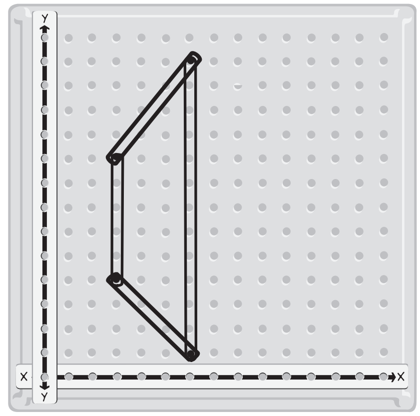


Area of triangle 12 sq. units

Area of triangle 15 sq. units

Area of trapezoid 27 sq. units

2.



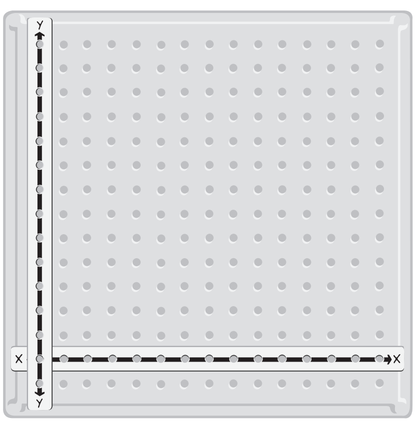
Area of triangle 7.5 sq. units

Area of triangle 18 sq. units

Area of trapezoid 25.5 sq. units

Using an XY Coordinate Pegboard, model a trapezoid with the given area. Sketch the model. Answer the questions.

3. 64 square units



What is the length of the short base? _____

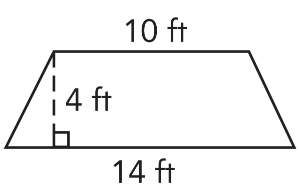
What is the length of the long base? _____

What is the height? _____

Check students' models; answers will vary depending on models.

Find the area of each trapezoid.

4.



48 ft²

5. bases, 4 in. and 6 in.
height, 5 in.

25 in.²

6. bases, 9 cm and 10 cm
height, 6 cm

57 cm²

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Answer Key

Challenge! How is finding the area of a trapezoid related to finding the area of two triangles, each with a base length equal to a base length of the trapezoid. Draw a picture to help.

Challenge: (Sample) One of the triangle's area is found using the formula $A = \frac{1}{2}h(b_1)$. The other triangle's area is found using the formula $A = \frac{1}{2}h(b_2)$. When the areas are combined, the Distributive Property shows the area is $A = \frac{1}{2}h(b_1 + b_2)$, which is the area formula for a trapezoid.

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LESSON 5

Geometry

Shapes in the Coordinate Plane

Objective

Draw shapes on a coordinate grid and describe their properties.

Common Core State Standards

- **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Many concepts of geometry can be illustrated on a coordinate grid. Viewing a shape on a coordinate grid helps students identify such properties of the shape as the number and lengths of its sides, the relationships between its sides, and in some cases the measures of its angles. Students who know these properties will more readily understand the formulas for area and perimeter.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What do you notice about the coordinates for the square?*
- **Ask:** *How does the coordinate grid help you describe the shapes?*
- Compare the square and the rhombus.
- Compare the square and the triangle.

Solve It

Reread the problem with students. Have students talk about the arrangement of the furniture using coordinates. Have them talk about how the grid helps Beth describe the placement and shape of each piece of furniture.

More Ideas

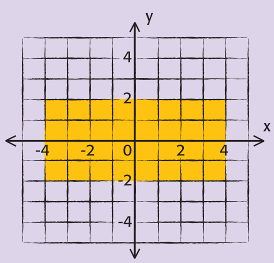
For other ways to teach about shapes in the coordinate plane—

- Have students make polygons on a Geoboard and describe the shapes using coordinates. Ask them to compare the lengths of the sides and the angles and the relationships between the sides of each figure as they describe it.
- Use AngLegs® and a 4-Quadrant Graph (BLM 12) to make squares, rhombuses, rectangles, parallelograms, and triangles. Tell students to move the shapes around the four quadrants and name the coordinates of each new position.

Formative Assessment

Have students try the following problem.

If the shape is moved up two units, what coordinates describe the shape after the move?



A. Rhombus; (–2, 2), (–2, –2), (2, 2), and (2, –2)

B. Rhombus; (–4, 2), (–4, –2), (6, 2), and (6, –2)

C. Rectangle; (–4, 4), (–4, 0), (4, 4), and (4, 0)

D. Rectangle; (–2, 2), (–2, –2), (6, 2), and (6, –2)

Try It! 30 minutes | Groups of 4

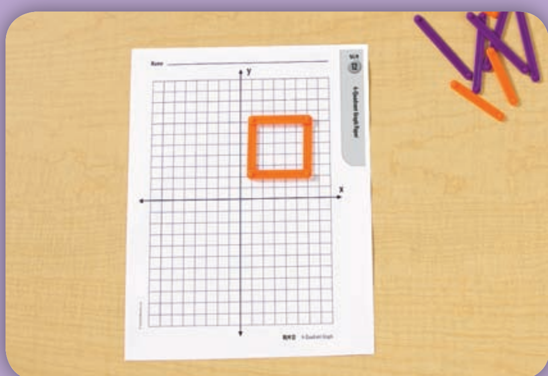
Here is a problem about drawing and describing shapes in the coordinate plane.

Beth is making a record of how the furniture on her patio is positioned. She divides the patio into quadrants and uses a 4-quadrant grid to show the placement of the furniture. She has a square chair, a triangular stool, and a table shaped like a rhombus. Show how the positions and shapes of the pieces of furniture might be represented on the grid.

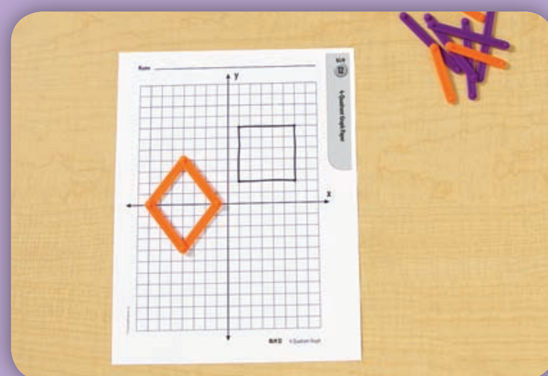
Introduce the problem. Then have students do the activity to solve the problem. Distribute AngLegs, graph paper, and pencils to students. Explain to students that the endpoints of the AngLegs are represented by the raised circles, not the extreme ends.

Materials

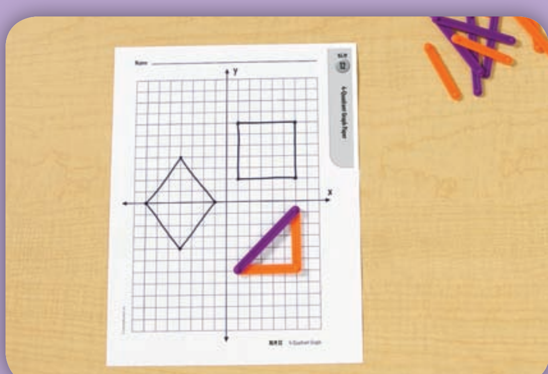
- AngLegs® (orange and purple only: one set per group)
- 4-Quadrant Graph Paper (BLM 12; 1 per group)
- pencils (1 per group)



1. Have students use an AngLegs piece to show a segment connecting points (1, 2) and (1, 7). **Say:** *This segment is one side of the square chair. Complete the chair so that it is positioned completely inside the first quadrant. Draw the shape.* Discuss with students how they found the additional coordinates.



2. Say: *One vertex of the rhombus-shaped table is at point (–4, 4) and another is at (–4, –4).* Have students connect the sides to make the rhombus. **Ask:** *What are the other coordinates of the rhombus?*



3. Have students use an AngLegs piece to show a segment connecting points (6, –1) and (6, –6). **Say:** *This segment is one side of the stool. Complete the stool so that there is a right angle at (6, –6).*

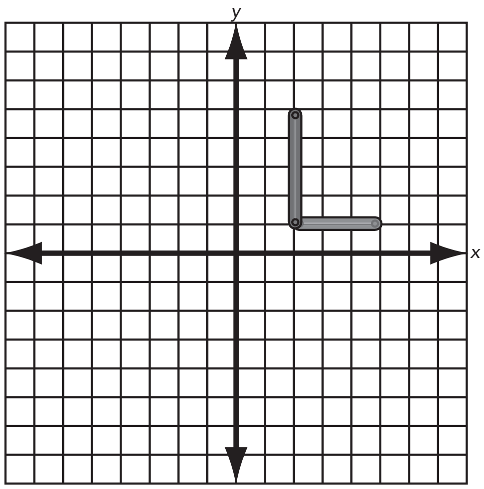
⚠ Look Out!

Some students might have difficulty determining how to connect the points correctly. If students use diagonals to stabilize the rhombus and square, remind them that only the sides of each shape are supposed to be represented; not the diagonals.

Use AngLegs® and graph paper to model each shape in a coordinate plane. Part of the shape is shown. Name the coordinates of the vertices that complete the shape.

(Check students' work.)

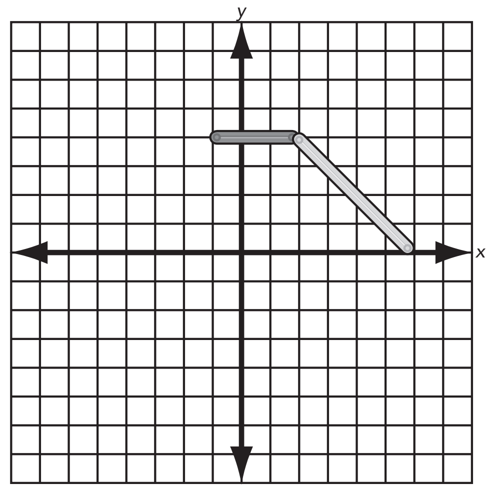
1. rectangle with vertices at (2, 1) and (2, 5)



The other vertices are at

(5, 1) and (5, 5) .

2. isosceles trapezoid with short base 3 units, long base 11 units

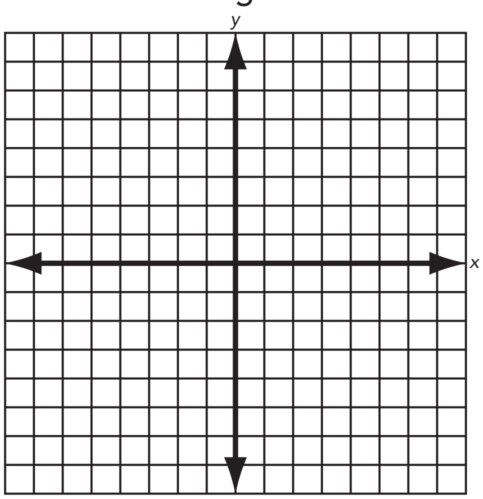


The vertices of the long base are

(–5, 0) and (6, 0) .

Using AngLegs, model each shape. Sketch the model. Name the vertices.

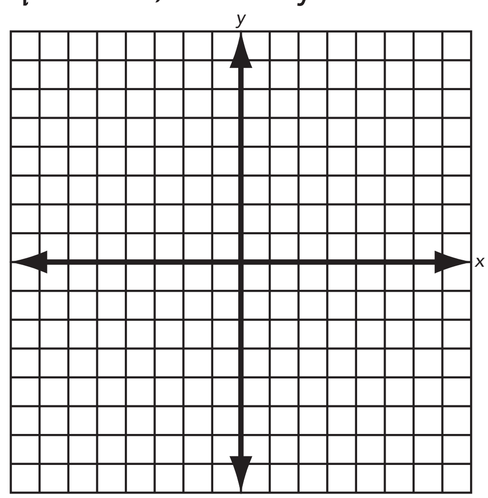
3. square in the second quadrant that has sides 5 units long



The vertices of the square are

Students' models will vary. .

4. rectangle in the third and fourth quadrants, 7 units by 4 units



The vertices of the rectangle are

Students' models will vary. .

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Answer Key

Challenge! If a rectangle has one vertex at (4, 4) and its opposite vertex is at (–5, –5), in what quadrants is the rectangle? Draw a picture to help.

Challenge: (Sample) The rectangle will be in all four quadrants.

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LESSON 6

Objective

Explore nets.

Common Core State Standards

- **6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Geometry

Nets

By constructing three-dimensional figures from two-dimensional representations, students see how plane shapes can be related to solid shapes. Identifying the faces, edges, and vertices on both polyhedrons and their nets helps students develop the visualization skills needed to find surface area.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How do you identify the faces in a net?*
- **Ask:** *How do you make a net for a rectangular box?*

Solve It

Reread the problem with students. Display a rectangular prism and explain that it represents a rectangular box. Have students sketch the solid and draw its net. Then have them write a paragraph explaining how Cameron can tell which cardboard piece is a net for a rectangular box.

More Ideas

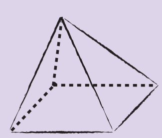
For other ways to teach about nets—

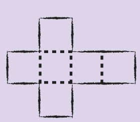
- Have students use Snap Cubes® to build rectangular prisms. Guide them to trace a face; then, without lifting the solid, turn it and trace a different face. Be sure students understand that the faces share edges but do not overlap. Have students cut out the nets and construct the prisms.
- Show students a Relational GeoSolids® rectangular prism. Review what a net is and challenge students to make different nets for the prism.

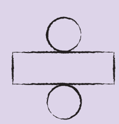
Formative Assessment

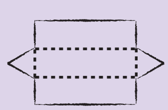
Have students try the following problem.

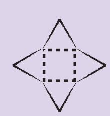
What net can be folded to make this solid?



A. 

B. 

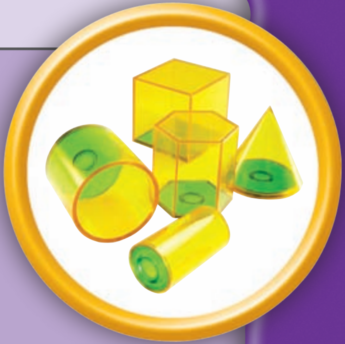
C. 

D. 

Try It! 30 minutes | Groups of 4

Here is a problem about nets.

Cameron works for a shipping company. Each shipping carton is stored as a flat cardboard piece that can be assembled when needed. What might the cardboard piece for a rectangular box look like?



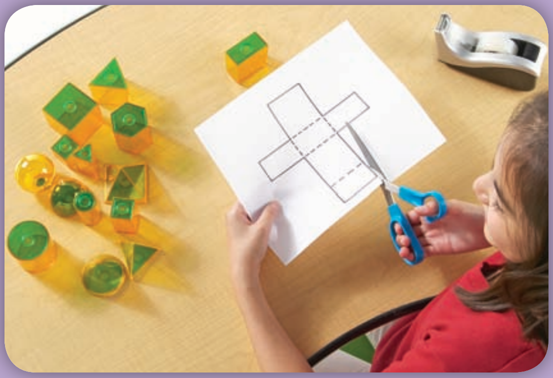
Introduce the problem. Then have students do the activity to solve the problem. Distribute solids, net patterns, paper, pencils, scissors, and tape to students. Explain that a net is a pattern that can be folded to make a three-dimensional object.

Materials

- Relational GeoSolids® (one set per group or two groups can share a set)
- Net Pattern (BLM 13; 1 per group)
- paper (2 sheets per group)
- pencils (1 per group)
- scissors (1 per group)
- tape (1 per group)

1. Have students examine the solids and describe how the shapes differ from two-dimensional shapes, such as rectangles. Display a cube and point out a face, an edge, and a vertex, defining each. **Say:** *Find a solid that has 12 vertices.*

2. Have students look at the net pattern and identify the faces and their shapes, the edges, and the vertices in the net. **Ask:** *Which solid figure has faces that are the same shape as the shapes in the net?* Have students cut out the net and fold it to make a solid.



3. Have students use one of the two rectangular prisms to make a net for a rectangular box. **Say:** *Cut and build your rectangular box from your net.*

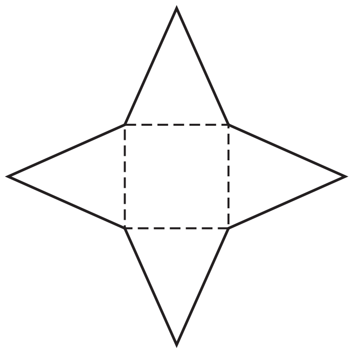
⚠ Look Out!

Some students may be inclined to cut out the faces and tape them together to form a solid. Point out that while this method can be used to create a three-dimensional shape, a net is a single piece that can be folded to make the shape. Instruct students to cut only along the outside of the plane figure.

Answer Key

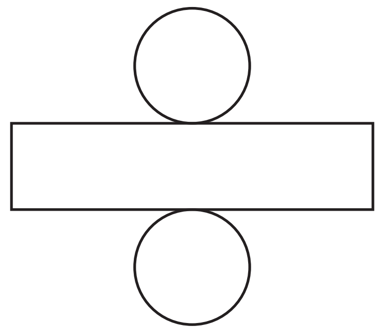
Use Relational GeoSolids to identify the solid for each net that is shown. Name the solid. (Check students' work.)

1.



square pyramid

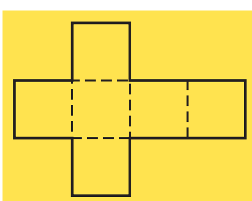
2.



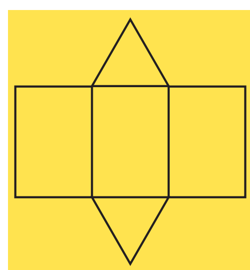
cylinder

Use Relational GeoSolids to help you draw a net for each solid. Sketch the net.

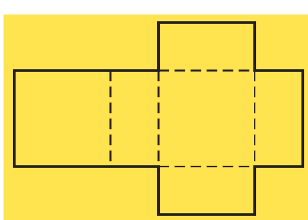
3. cube



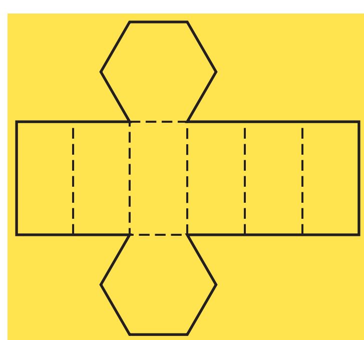
4. triangular prism



5. rectangular prism



6. hexagonal prism



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Answer Key

Challenge! How many different nets can you draw for a cube?

Challenge: Check students' drawings. There are 11 different nets for a cube.

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LESSON 7

Objective

Find the surface area of a rectangular solid.

Common Core State Standards

- **6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Geometry

Surface Area of a Rectangular Solid

Using a model to explore and find surface area helps students distinguish surface area from volume and helps them visualize the dimensions of each face of a solid. In this lesson, students build a rectangular solid and use it to find surface area and to develop a formula for finding surface area.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How are all rectangular solids alike? How can they be different?*
- **Ask:** *What does each square unit in the model represent?*
- **Say:** *Pairs of faces on a rectangular solid are congruent.* **Ask:** *How can you use this fact to help you find the total surface area?*
- **Ask:** *How can a net help you find the surface area of a solid?*

Solve It

Reread the problem with students. Have them explain how to find the surface area of a rectangular solid. Have students cut out their net and fold and tape it to make the prism.

More Ideas

For other ways to teach about surface area of rectangular solids—

- Students work in pairs, using Centimeter Cubes to build two models of rectangular solids in which the dimensions of one are twice the dimensions of the other, such as 4 × 6 × 2 and 2 × 3 × 1. Have students find and compare the surface areas of the solids.
- Have students make nets of the rectangular prisms in a set of Relational GeoSolids®. Students can draw a unit grid on each face of a net and count squares to find the total surface area in arbitrary square units.

Formative Assessment

Have students try the following problem.

How much construction paper does Bryant need to cover a pencil box that is 14 inches long, 6 inches wide, and 1 inch high?

A. 84 square inches

B. 188 square inches

C. 202 square inches

D. 208 square inches

Try It! 40 Minutes | Groups of 4

Here is a problem about the surface area of a rectangular solid.

Teri plans to cover all sides of a jewelry box in fabric. How much fabric does she need if the length of the jewelry box is 6 inches, the width is 4 inches, and the height is 2 inches? Write a formula for the surface area of a rectangular solid.

Introduce the problem. Then have students do the activity to solve the problem. Distribute Snap Cubes, paper, pencils, scissors, and tape to students. **Say:** *The surface area of a rectangular solid is the sum of the areas of its faces.*

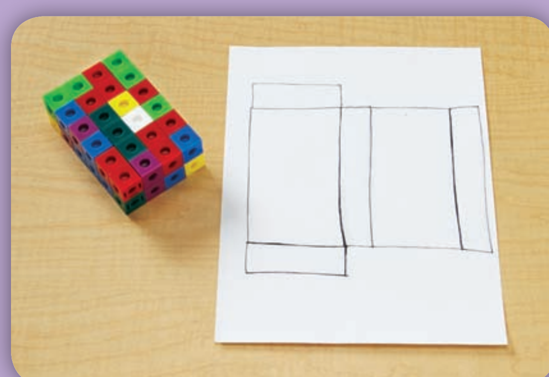
Materials

- Snap Cubes® (50 per group)
- paper (2 sheets per group)
- pencils (1 per group)
- scissors (1 per group)
- tape (1 per group)

1. Say: *Use Snap Cubes to build a rectangular solid that is 6 units long, 4 units wide, and 2 units high.* **Ask:** *How many faces does the solid have? How many pairs of faces are congruent?* Students should see that there are six faces and three pairs of congruent faces.

2. Ask: *How can you find the areas of the faces of the rectangular solid?* Guide students to see that they can count the number of square units on the faces or they can multiply the dimensions of the faces. **Say:** *Record the areas of the faces. Add the areas of the faces to get the total surface area.*

3. Say: *You can write a formula for surface area.* **Ask:** *Which dimensions do you multiply together for each pair of faces?* Encourage students to use their models to see that they multiply length by width, length by height, and width by height. Help them write the formula.



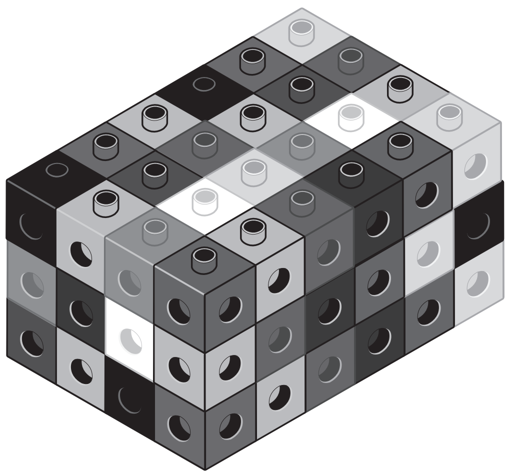
4. To enhance students' visual sense of the problem, have students draw a net of the solid. Encourage students to make the net actual size by tracing the solid's faces. **Ask:** *How does the net help you visualize the problem?* Students can draw the unit squares on the faces.

Answer Key

Use Snap Cubes to build the rectangular solid. Find the surface area.

(Check students' work.)

1.



area of top surface 24 square units

area of bottom surface 24 square units

area of right side surface 18 square units

area of left side surface 18 square units

area of front surface 12 square units

area of back surface 12 square units

Total surface area 108 square units

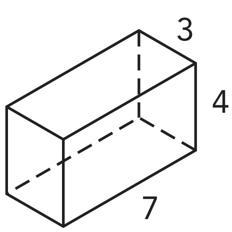
Using Snap Cubes, build the solid with the given dimensions. Sketch the model. Find the surface area.

2. length: 5 units, width: 4 units, and height: 5 units

130 square units

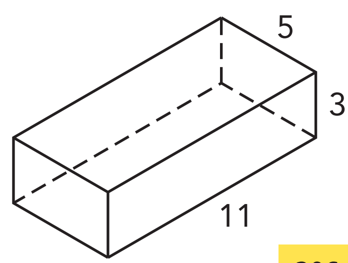
Find the surface area of each rectangular solid.

3.



122 square units

4.



206 square units

5. length: 6 units
width: 4 units
height: 1 unit

68 square units

6. length: 9 units
width: 2 units
height: 2 units

80 square units

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Answer Key

Challenge! How many faces does a rectangular solid have? How are these faces used to find the surface area of the solid?

Challenge: (Sample) 6; Find the area of each face. Add the areas of all six faces to find the total surface area.

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