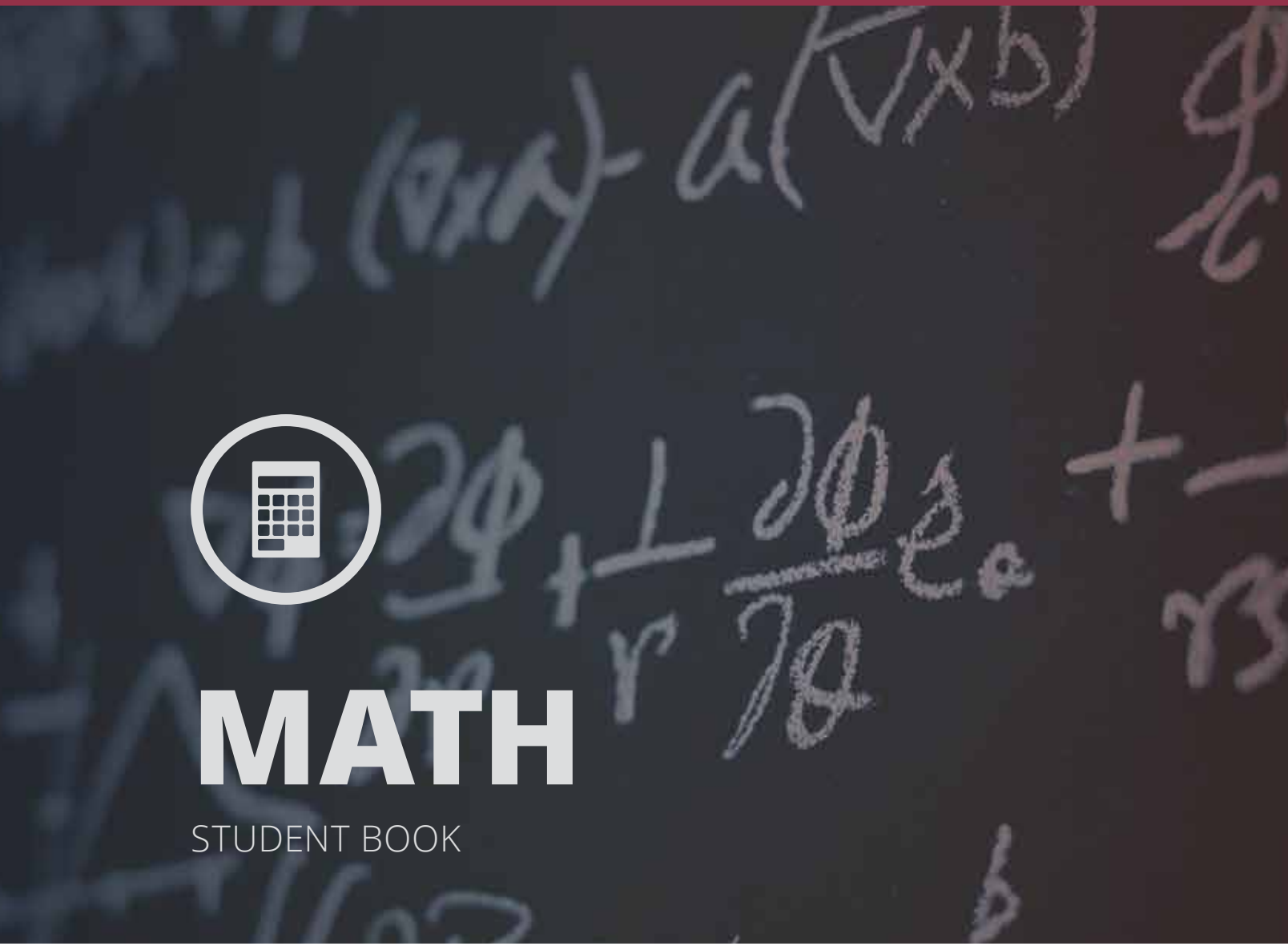
LIFEPAC®



MATH

STUDENT BOOK

9th Grade | Unit 1

Alpha Omega
PUBLICATIONS

Math 901

Variables and Numbers

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Variables and Numbers

INTRODUCTION

This LIFEPAC® is your introduction to a system of mathematics unlike the arithmetic you learned in the elementary grades. In arithmetic you were taught the rules that govern the four operations of the system—addition, subtraction, multiplication, and division; and you were told which operation to perform on a given set of numbers to get the answer. Here are typical examples of exercises in arithmetic: 4 + 3 = 7, 7 – 2 = 5, 13 x 4 = 52, 12 ÷ 3 = 4.

Algebra, like geometry, trigonometry, and calculus, is another of several mathematical *systems*. Like arithmetic, it has its own operating rules. Unlike arithmetic, algebra often requires you to find the value of one of the numbers—the *unknown*—in an exercise. Sometimes, you will have to decide for yourself what operation to use, and sometimes several operations will be used. Toward the end of this LIFEPAC, you will learn how to apply the arithmetic operations to numbers less than zero—the negative numbers.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify bases, exponents, constants, variables, numerical coefficients, terms, sums, and products.
2. Simplify algebraic expressions when possible.
3. Evaluate algebraic expressions.
4. Translate algebraic expressions.
5. Perform operations with signed numbers.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. EXPRESSIONS

The expression 8 + 3 is a *numerical expression*: *numerical* because it consists of numbers; *expression* because it expresses an operation, in this case addition.

In algebra letters of the alphabet are used to represent numbers. These letters are referred to either as *unknowns* or as *variables*. An expression that contains a variable, such as $n + 3$, is an *algebraic expression*. Learning to handle algebraic expressions is the first step in this new system of mathematics. You will have an opportunity in this section to review and practice basic number skills and then to apply those skills in simplifying expressions by the distributive property.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify bases, exponents, constants, variables, numerical coefficients, terms, sums, and products.
2. Simplify algebraic expressions when possible.
3. Evaluate algebraic expressions.
4. Translate algebraic expressions.

VARIABLES

If expressions, whether numerical or algebraic, imply addition, they are called *sums*; if they imply subtraction, they are called *differences*; if multiplication, *products*; and if division, *quotients*. These four operations will now be used in evaluating expressions.

SUMS AND DIFFERENCES

In the expression $n + 3$, n and 3 are *addends*. Since we have inserted the plus sign between the letter n and the number 3, the expression is called an *indicated sum*. Its value cannot be determined until we know the value of n.

The expression $n - 3$ means that 3 is to be subtracted from n. Likewise, $3 - n$ means that n is to be subtracted from 3. $n - 3$ is called an *indicated difference*. The expressions $n - 3$ and $3 - n$ are not necessarily equal, because subtraction is an *ordered* operation. We see that $8 - 3$ cannot be $3 - 8$. The *differences* are different.

In an algebraic expression, the letter that represents a number is called a *variable*. In the expression $n + 8$, n is the variable and 8 is the *constant*.

Here are some other models of sums and differences.

$$5 - y,\ x + 6,\ A + 10,\ A + B,\ x + y,\ x - y$$

Simplify. Work from left to right and perform any operation in parentheses first.

Model: 9 + 12 – 3 = 21 – 3 = 18
(2 + 5) – 4 = 7 – 4 = 3

1.1	9 + 6	_____	**1.2**	8 + 13	_____
1.3	5 + 22	_____	**1.4**	17 + 16	_____
1.5	32 + 43	_____	**1.6**	9 + 5 + 4	_____
1.7	3 + 8 + 4	_____	**1.8**	10 + 15 + 4	_____
1.9	17 + 18 + 5	_____	**1.10**	14 + 13 + 7	_____
1.11	10 – 6 + 8	_____	**1.12**	15 – 4 + 1	_____
1.13	17 – 3 – 4	_____	**1.14**	13 – 8 + 10	_____
1.15	28 + 4 – 10	_____	**1.16**	5 + (6 – 4)	_____
1.17	10 + (3 – 2)	_____	**1.18**	29 – (7 + 2)	_____
1.19	(13 + 2) – 8	_____	**1.20**	(50 + 5) – 11	_____

Write the meaning of each of the following expressions.

Model: $x + 10$ The sum of some number x and 10.

1.21	$n + 5$	_____
1.22	$n - 5$	_____
1.23	$x + 8$	_____
1.24	$x - 8$	_____

1.25 $8 - x$ _____

1.26 $5 - y$ _____

1.27 $x + (5 + 7)$ _____

1.28 $x - (8 + 2)$ _____

1.29 $x + (8 - 2)$ _____

1.30 $x + x$ _____

Identify the variable and constant in each of the following expressions and tell if it is a sum, a difference, or neither.

		Variable	Constant	Operation
Model:	$x - 8$	a. *x*	b. 8	c. difference
1.31	$6 + y$	a._____	b._____	c._____
1.32	$N - 8$	a._____	b._____	c._____
1.33	A	a._____	b._____	c._____
1.34	$B - 3$	a._____	b._____	c._____
1.35	$C + 10 + 12$	a._____	b._____	c._____

Write an algebraic expression of each of the following statements.

1.36 The sum of *n* and 6. _____

1.37 The difference of 8 and *n.* _____

1.38 The difference of *n* and 10. _____

1.39 The sum of *n* and itself. _____

1.40 The sum of *n* and the sum of 8 and 6. _____

Sums like 8 + 3 may be written as 3 + 8.

The sum 11 is the same in either case. The ability to interchange addends is called the *commutative property* of addition.

Also, sums like 4 + 2 + 7 may be obtained from (4 + 2) + 7 or from 4 + (2 + 7).

Model: Simplify $3 + x + 7$.

$$\begin{aligned} 3 + x + 7 &= x + 3 + 7 \\ &= x + (3 + 7) \\ &= x + 10 \end{aligned}$$

The sum 13 is the same in either case. The ability to change the grouping of the addends is called the *associative property* of addition.

These two properties can be used to simplify expressions.

Simplify

1.41 $x + 7 + 8$

_____

1.42 $7 + x + 3$

_____

1.43 $9 + 7 + n$

_____

1.44 $x + 15 - 4$

_____

1.45 $(20 + 2) + r$

_____

1.46 $8 + r - 4$

_____

1.47 $15 + x + 10 - 4$

_____

1.48 $(15 - 10) + n$

_____

1.49 $5 + n + (15 - 2)$

_____

1.50 1.5 + 3.82

_____

1.51 17.25 + 3.9

_____

1.52 19.62 + 8.33 + 5.7

_____

1.53 1.005 + 3.54

_____

1.54 73.05 + 8.006

_____

1.55 15.63 + 7.956 + 82.735

_____

1.56 25.63 – 8.23

_____

1.57 73.543 – 23.683

_____

1.58 28.543 – 14.26 – 3.65

_____

1.59 x + 6.2 + 8.5

_____

1.60 7.5 + n + 9.63

_____

1.61 81.56 + n – 2.55

_____

1.62 7.95 – 3.86 + N

_____

1.63 22.6 + x – 11.3 + 1.2

_____

1.64 77.65 – 15.56 + x + 1.2

_____

PRODUCTS

The numerical expression 7 + 7 can be renamed several ways, one of which is 2 times 7. We wish to omit the (x) as a times sign. In algebra we will use the dot, 2 • 7, or the parentheses, (2)(7). Therefore, the product of 6 and 9 will be written as 6 • 9 or (6)(9). Likewise, if one of the *factors is literal* — the n in 7 times n — we will write the product as 7n. The dot or parentheses are not to be used when writing literal products.

- **Models:** 6 • 4, 6x, 5n, 15r, r17, A15

Product expressions such as r • 17 and A • 15 are to be written with the constant preceding the variable, 17r and 15A. The constant preceding the variable in a product is called a *numerical coefficient*.

Find the product of each of the following expressions.

1.65	6 • 5	_____	**1.66**	8 • 4	_____
1.67	7 • 6	_____	**1.68**	9 • 8	_____
1.69	4 • 10	_____	**1.70**	(5)(10)	_____
1.71	(15)(12)	_____	**1.72**	(3)(50)	_____
1.73	(40)(5)	_____	**1.74**	(70)(20)	_____
1.75	6(15)	_____	**1.76**	7(22)	_____
1.77	8(15)	_____	**1.78**	16(30)	_____
1.79	10(23)	_____	**1.80**	5 • 7 • 8	_____
1.81	12 • 5 • 8	_____	**1.82**	3(4)(5)	_____
1.83	6(2)(5)	_____	**1.84**	15(2)(8)	_____

Name the numerical coefficient of each of the following expressions.

1.85	$6x$	_____	**1.86**	$5n$	_____
1.87	$22r$	_____	**1.88**	$16p$	_____
1.89	$13q$	_____	**1.90**	$8 \cdot 2N$	_____
1.91	$3 \cdot 2x$	_____	**1.92**	$7.2r$	_____
1.93	$9(14)P$	_____	**1.94**	$2(3)(6)q$	_____

In the operation 5 • 7, the product is the same if the expression is changed to 7 • 5. That is, 5 • 7 = 7 • 5. The ability to interchange factors is called the *commutative property* of multiplication. Also, the *associative property* of multiplication allows you to change the grouping of the factors.

These two properties can be used to simplify expressions. When more than one variable is used, the letters are to be written in alphabetical order.

Model 1: Rewrite $B \cdot 5 \cdot A$

$$\begin{aligned} B \cdot 5 \cdot A &= 5 \cdot B \cdot A \\ &= 5 \cdot (B \cdot A) \\ &= 5 \cdot (A \cdot B) \\ &= 5AB \end{aligned}$$

Model 2: Rewrite $7 \cdot K \cdot 5 \cdot H$

$7 \cdot K \cdot 5 \cdot H$ may be rewritten as $(7 \cdot 5)(H \cdot K)$ using the commutative and associative properties; thus, the simplified form is $35HK$.

Simplify. Remember: When more than one variable is used, the letters are to be written in alphabetical order. Also, no dots are to be shown in the final answers.

1.95 $6 \cdot x \cdot 7$ _____

1.96 $5 \cdot P \cdot 2$ _____

1.97 $3 \cdot S \cdot R$ _____

1.98 $8 \cdot x \cdot 2 \cdot y$ _____

1.99 $a \cdot c \cdot 2 \cdot 5$ _____

1.100 $C \cdot 5 \cdot 2 \cdot A$ _____

1.101 $4 \cdot Q \cdot 2 \cdot P$ _____

1.102 $10 \cdot K \cdot 2$ _____

Write the meaning of each of the following expressions.

Model: $4A$ The product of 4 and some number.

Model: $10N - 2$ The difference between ten times some number and 2.

1.103 $7n$ _____

1.104 $6P$ _____

1.105 $8N + 5$ _____

1.106 $7 + 2x$ _____

1.107 $12x - 10$ _____

1.108 $52 - 25x$ _____

EXPONENTS

The numerical expression 5 times 5 may be written as 5^2. The 2 is called an exponent. The exponent is a counter for the number of repeated factors.
Thus $6 \cdot 6 = 6^2$ and $8 \cdot 8 \cdot 8 = 8^3$.

In the case of literal expressions, we have $x \cdot x = x^2$ and $A \cdot A \cdot A = A^3$. Conversely, x^3 means $x \cdot x \cdot x$, or three factors of x.

Models:

$x^2 = x \cdot x$	$P^2 = P \cdot P$	$(ab)^2 = ab \cdot ab$
$x^3 = x \cdot x \cdot x$	$P^3 = P \cdot P \cdot P$	$(ab)^3 = ab \cdot ab \cdot ab$
$x^4 = x \cdot x \cdot x \cdot x$	$P^4 = P \cdot P \cdot P \cdot P$	$(ab)^4 = ab \cdot ab \cdot ab \cdot ab$

x^2 is read, "The square of x" or "x squared."
x^3 is read, "The cube of x" or "x cubed."
x^4 is read, "The fourth power of x" or "x to the fourth."

x^n is an *indicated power*. x is called the base, and n is the *exponent* of the base.

In each case, identify the base and the exponent of the indicated power.

Model: 3^8 base = 3 exponent = 8

		Base	Exponent			Base	Exponent
1.109	2^6			**1.110**	3^9		
1.111	5^{10}			**1.112**	8^3		
1.113	x^6			**1.114**	y^5		
1.115	7^n			**1.116**	9^P		
1.117	15^{x+1}			**1.118**	10^{3x-1}		

Write each of the following expressions in product form.

Model: $A^3 = A \cdot A \cdot A$

1.119	6^3	=		**1.120**	7^4	=	
1.121	x^2	=		**1.122**	y^5	=	
1.123	3^3	=		**1.124**	1^4	=	
1.125	2^5	=		**1.125**	$(\frac{1}{2})^3$	=	
1.127	$(2.5)^2$	=		**1.128**	$(.01)^4$	=	

Simplify each of the following expressions.

Model: $3^2 = 3 \cdot 3 = 9$

1.129 2^3 _____

1.130 4^2 _____

1.131 5^3 _____

1.132 3^5 _____

1.133 10^3 _____

1.134 10^4 _____

1.135 17^2 _____

1.136 20^2 _____

1.137 7^3 _____

Circle the larger number in each pair.

Model: 1^2, (2²) since $1^2 = 1 \cdot 1 = 1$ and $2^2 = 2 \cdot 2 = 4$

1.138 2^5 or 5^2

1.139 5^2 or 5^3

1.140 3^2 or 2^3

1.141 3^3 or 2^5

1.142 $(\frac{1}{2})^3$ or $(\frac{1}{3})^2$

1.143 $(.2)^2$ or $(.3)^2$

Write the following products in exponential form.

Model: $A \cdot A \cdot A = A^3$

1.144 $x \cdot x \cdot x \cdot x$ _____

1.145 $B \cdot B \cdot B \cdot B \cdot B$ _____

1.146 $P \cdot P$ _____

1.147 $N \cdot N \cdot N$ _____

1.148 $A \cdot B \cdot A \cdot B$ _____

1.149 $C \cdot d \cdot C \cdot d \cdot C$ _____

1.150 $xyyx$ _____

1.151 $PPQQQ$ _____

1.152 $abcabcabc$ _____

1.153 $xyzxyz$ _____

EVALUATING EXPRESSIONS

Expressions containing variables can be *evaluated* when numerical values are given to the variables.

Model: In the expression $A + 4$
If $A = 10$, then $10 + 4 = 14$.
If $A = 75$, then $75 + 4 = 79$.

To evaluate expressions involving more than one operation, you are to use the following *order of operations*: working left-to-right, do any powers first, then do any multiplications or divisions,and finally do any additions or subtractions.

Model 1: Evaluate $5x - 10$ for $x = 30$

Solution: Replace x with 30 and evaluate.

	$5x - 10$
Replace x with 30	$5 \cdot 30 - 10$
Multiply	$150 - 10$
Subtract	140

Model 2: Evaluate $A^2 + 2A + 5$ for $A = 4$.

Solution:

$A^2 + 2A + 5$
$4^2 + 2 \cdot 4 + 5$
$16 + 8 + 5$
$24 + 5$
29

Evaluate.

1.154	$a + 6$	for $a = 10$	_____
1.155	$16 - B$	for $B = 2$	_____
1.156	B^2	for $B = 9$	_____
1.157	A^2	for $A = 2.3$	_____
1.158	$x^2 + 2$	for $x = 5.1$	_____

Evaluate for $a = 2$, $b = 3$, and $c = 4$.

1.159	$5a$	_____	**1.160**	$2b$	_____
1.161	$4c$	_____	**1.162**	a^2	_____
1.163	ab	_____	**1.164**	$a + b$	_____
1.165	$a + b + c$	_____	**1.166**	$ab + c$	_____
1.167	$a + bc$	_____	**1.168**	abc	_____
1.169	a^2b	_____	**1.170**	$a^2\ b^2\ c^2$	_____
1.171	$2a - b$	_____	**1.172**	$c - a$	_____
1.173	$b - a$	_____	**1.174**	$3a^2$	_____
1.175	$3ab$	_____	**1.176**	$4ab^2$	_____
1.177	$(a + b)^2$	_____	**1.178**	$(b + c)^3$	_____
1.179	$a^2b^2 + b^2c^2$	_____	**1.180**	$a + b^2$	_____
1.181	$4(a + b)$	_____	**1.182**	$3(a + b)^2$	_____
1.183	$(b + c)^2$	_____	**1.184**	$(a + b + c)^2$	_____
1.185	$(b - a)^2$	_____	**1.186**	$3a^2 + 4b^2$	_____
1.187	$5a^3 + 2b$	_____	**1.188**	$3(a + b + c)^2$	_____

Write the algebraic expression for each of the following sentences. Use any letters you wish.

1.189 Seven added to some number. _____

1.190 The square of some number. _____

1.191 Two times the cube of some number. _____

1.192 The difference between the square of a number and 10. _____

1.193 The difference between the squares of two numbers. _____

Write in words the meaning of each expression.

1.194 x^3 _____

1.195 $x^2 + 2$ _____

1.196 $3x^2 - 4$ _____

1.197 $A^2 - B^2$ _____

1.198 $3A + 4B^2$ _____

NUMBER SKILLS

Success with algebra is directly related to your understanding of basic skills in arithmetic. Included in this section is practice in adding, subtracting, multiplying, and dividing. Practice with fractions and percents is also included.

INTEGERS AND DECIMALS

Addition, subtraction, multiplication, and division of integers and decimals are building-blocks skills for working with fractions and percents.

Add.

1.199
$$\begin{array}{r} 65 \\ +\ 84 \\ \hline \end{array}$$

1.200
$$\begin{array}{r} 73 \\ +\ 58 \\ \hline \end{array}$$

1.201
$$\begin{array}{r} 96 \\ 82 \\ +\ 45 \\ \hline \end{array}$$

1.202
$$\begin{array}{r} 57 \\ 29 \\ +\ 78 \\ \hline \end{array}$$

1.203
$$\begin{array}{r} 65 \\ 43 \\ 92 \\ +\ 75 \\ \hline \end{array}$$

1.204
$$\begin{array}{r} 623 \\ 532 \\ +\ 481 \\ \hline \end{array}$$

1.205
$$\begin{array}{r} 962 \\ 853 \\ +\ 451 \\ \hline \end{array}$$

1.206
$$\begin{array}{r} 7.26 \\ 5.38 \\ +\ 62.73 \\ \hline \end{array}$$

1.207
$$\begin{array}{r} 5.06 \\ 12.55 \\ +\ 1.075 \\ \hline \end{array}$$

1.208
$$\begin{array}{r} 52.631 \\ 7.05 \\ +\ 9.006 \\ \hline \end{array}$$

Subtract.

1.209
$$\begin{array}{r} 72 \\ -\ 58 \\ \hline \end{array}$$

1.210
$$\begin{array}{r} 59 \\ -\ 28 \\ \hline \end{array}$$

1.211
$$\begin{array}{r} 73 \\ -\ 48 \\ \hline \end{array}$$

1.212
$$\begin{array}{r} 92 \\ -\ 79 \\ \hline \end{array}$$

1.213
$$\begin{array}{r} 523 \\ -\ 99 \\ \hline \end{array}$$

1.214
$$\begin{array}{r} 6521 \\ -\ 438 \\ \hline \end{array}$$

1.215
$$\begin{array}{r} 5431 \\ -\ 3413 \\ \hline \end{array}$$

1.216
$$\begin{array}{r} 5.83 \\ -\ 2.96 \\ \hline \end{array}$$

1.217
$$\begin{array}{r} 93.056 \\ -\ 43.685 \\ \hline \end{array}$$

1.218
$$\begin{array}{r} 1.2306 \\ -\ 0.9615 \\ \hline \end{array}$$

Multiply. Show your work.

1.219 633×4

1.220 586×28

1.221 $7.23 \times .02$

1.222 52.63×1.54

1.223 $0.056 \times .73$

1.224 929×29

1.225 5263×251

1.226 4.356×27.3

1.227 66.28×5.84

1.228 10.05×1.06

Divide. Show your work.

1.229 $\frac{659}{8}$

1.230 $\frac{732}{2}$

1.231 $\frac{564}{4}$

1.232 $\frac{121}{11}$

1.233 $\frac{144}{6}$

1.234 $8\overline{)2563}$

1.235 $9\overline{)28.54}$

1.236 $7\overline{)5280}$

1.237 $63\overline{)596}$

1.238 $85\overline{)726.5}$

1.239 $9.2\overline{)52.063}$

FRACTIONS

Remember that fractions must have common denominators to be added or subtracted. Products of fractions are found by multiplying numerator by numerator and denominator by denominator. Quotients of fractions are found by using the reciprocal of the divisor and then multiplying. All results are to be reduced to lowest terms.

Models:

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

$3\frac{1}{4} - 2\frac{1}{5} = 3\frac{5}{20} - 2\frac{4}{20} = 1\frac{1}{20}$

$\frac{3}{5} \cdot \frac{2}{6} = \frac{6}{30} = \frac{1}{5}$

$2\frac{1}{2} \div 1\frac{3}{7} = \frac{5}{2} \div \frac{10}{7} = \frac{5}{2} \cdot \frac{7}{10} = \frac{35}{20} = \frac{7}{4}$ or $1\frac{3}{4}$

Add. Show your work.

1.240 $\frac{2}{3} + \frac{3}{8} =$

1.241 $\frac{5}{4} + \frac{7}{9} =$

1.242 $\frac{15}{7} + \frac{2}{3} + \frac{1}{6} =$

1.243 $1\frac{2}{3} + 5\frac{3}{8} =$

1.244 $10\frac{3}{7} + 19\frac{5}{9} =$

1.245 $1\frac{1}{2} + 5\frac{3}{4} =$

Subtract. Show your work.

1.246 $\frac{6}{11} - \frac{4}{11} =$

1.247 $\frac{9}{32} - \frac{1}{16} =$

1.248 $\frac{13}{16} - \frac{3}{8} =$

1.249 $5\frac{1}{6} - 2\frac{2}{3} =$

2.250 $7\frac{9}{16} - 5\frac{1}{5} =$

1.251 $2\frac{5}{8} - 1\frac{3}{8} =$

Multiply. Show your work.

1.252 $\frac{2}{3} \cdot \frac{5}{8} =$

1.253 $\frac{5}{17} \cdot \frac{3}{8} =$

1.254 $\frac{4}{11} \cdot \frac{10}{8} =$

1.255 $7\frac{1}{8} \cdot 5\frac{2}{3} =$

1.256 $15\frac{4}{9} \cdot 3\frac{1}{5} =$

Divide. Show your work.

1.257 $\frac{2}{3} \div \frac{4}{9} =$

1.258 $\frac{1}{3} \div \frac{3}{8} =$

1.259 $\frac{4}{11} \div \frac{4}{9} =$

1.260 $8\frac{1}{2} \div 2\frac{1}{4} =$

PERCENT

Skills with percent include converting percents to decimals, decimals to percents, and fractions to percent. To change percents to decimals, divide the percent by 100; to change decimals to percent, multiply the decimal by 100 and include the % sign. To change fractions to percent, multiply the fraction by 100, reduce the result, and include the % sign.

Models:

Change 34% to a decimal number.
$34\% = 34 \div 100 = 0.34$

Change 0.19 to a percent.
$0.19 = 0.19 \cdot 100\% = 19\%$

Change $\frac{3}{4}$ to a percent.

$\frac{3}{4} = \frac{3}{4} \cdot 100\% = \frac{300}{4}\% = 75\%$

Change each percent to a decimal.

1.261 16% _____

1.262 22% _____

1.263 0.5% _____

1.264 302% _____

1.265 1.6% _____

Change each decimal number to a percent.

1.266 0.15 _____

1.267 0.06 _____

1.268 1.05 _____

1.269 32 _____

1.270 0.0075 _____

Change each fraction to a percent. Show your work.

1.271 $\frac{1}{2}$ _____

1.272 $\frac{3}{8}$ _____

1.273 $\frac{5}{20}$ _____

1.274 $\frac{4}{12}$ _____

1.275 $\frac{15}{50}$ _____

Write the required quantities. Show your work.

1.276 15% of 20 _____

1.277 13% of 50 _____

1.278 72% of 653 _____

1.279 35% of 70 _____

1.280 30 is what % of 60 _____

1.281 66 is what % of 150 _____

1.282 70 is 30% of what number _____

1.283 90 is 50% of what number _____

THE DISTRIBUTIVE PROPERTY

Algebraic expressions often use parentheses to group a sum or difference of two or more numbers. The *distributive property* is the rule that may be used to evaluate the product of a number and an expression in parentheses.

NUMBERS

The expression 7(4 + 5) means to multiply seven by the sum of 4 and 5. If we add the 4 and 5 first we have:

$$7(4 + 5) = 7(9) = 63.$$

However, we may multiply another way:

$$7(4 + 5) = 7 \cdot 4 + 7 \cdot 5$$

$$= 28 + 35$$

$$= 63$$

Both methods lead to the same result. The latter method applies the *distributive property*.

Another model of the distributive property is:

$$8(5 + 6) = 8 \cdot 5 + 8 \cdot 6$$

$$= 40 + 48$$

$$= 88$$

To verify that this method of multiplication is correct, add the 5 and 6 first, then multiply:

$$8(5 + 6) = 8(11) = 88.$$

The distributive property stated formally is:

DISTRIBUTIVE PROPERTY

For numbers a, b, c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

Models:

$$\begin{aligned} 6(3 + 2) &= 6 \cdot 3 + 6 \cdot 2 \\ &= 18 + 12 \\ &= 30 \end{aligned}$$

$$\begin{aligned} 7(5 - 2) &= 7 \cdot 5 - 7 \cdot 2 \\ &= 35 - 14 \\ &= 21 \end{aligned}$$

$$\begin{aligned} 3(20 + 2) &= 3 \cdot 20 + 3 \cdot 2 \\ &= 60 + 6 \\ &= 66 \end{aligned}$$

$$\begin{aligned} 12(100 - 1) &= 12 \cdot 100 - 12 \cdot 1 \\ &= 1200 - 12 \\ &= 1188 \end{aligned}$$

Use the distributive property and simplify. Show your work as in the preceding models.

1.284 8(4 + 3)

_____

1.285 9(8 + 2)

_____

1.286 15(5 + 2)

_____

1.287 17(4 + 1)

_____

1.288 13(5 + 4)

_____

1.289 20(2 + 3)

_____

1.290 6.5(5 + 1)

_____

1.291 8.6(3.2 + 4.6)

_____

1.292 7(2 + 3 + 4)

_____

1.293 5(5 + 4 + 1)

_____

1.294 6(3 + 2 + 5)

_____

1.295 10(1 + 3 + 5)

_____

$x + y$ is called an *indicated sum*,
$x \cdot y$ is called an *indicated product*, and
$x - y$ is called an *indicated difference*.

Models:

Indicated sums

$6 + 8$
$m + 2$
$d + (6 - 3)$
$(7 + m) + n$
$4 \cdot 5 + 4 \cdot 7$

Indicated products

$6 \cdot 8$
$2j$
$r(6 + 5)$
$3p \cdot 4q$
$4(5 + 7)$

Indicated differences

$5 - 3$
$2 - s$
$y - (4 + 3)$
$5m - 4n$
$3 \cdot 7 - 3 \cdot 2$

Since the distributive property states that $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$, we may also write $ab + ac = a(b + c)$ and $ab - ac = a(b - c)$.

$ab + ac$ is an indicated sum, and $a(b + c)$ an indicated product.

$ab - ac$ is an indicated difference, and

$a(b - c)$ is an indicated product.

Model: Change $6 \cdot 2 + 6 \cdot 5$ to an indicated product.

Solution: Since $a \cdot b + a \cdot c = a(b + c)$ and in our problem $a = 6$, $b = 2$, and $c = 5$, we have $6 \cdot 2 + 6 \cdot 5 = 6(2 + 5)$.

This is an example of using the distributive property to change an indicated sum to an indicated product.

Change the following indicated sums to indicated products as in the preceding model. (Do not find the answer.)

1.296	$5 \cdot 20 + 5 \cdot 3$	**1.297**	$8 \cdot 16 + 8 \cdot 4$	**1.298**	$9 \cdot 7 + 9 \cdot 8$
	_____		_____		_____
1.299	$6 \cdot 5 + 6 \cdot 8$	**1.300**	$15 \cdot 4 + 15 \cdot 10$	**1.301**	$9 \cdot 10 + 9 \cdot 5$
	_____		_____		_____
1.302	$5 \cdot 8 + 9 \cdot 5$	**1.303**	$4 \cdot 7 + 8 \cdot 7$	**1.304**	$3 \cdot 10 + 20 \cdot 3$
	_____		_____		_____

One numerical application of the distributive property is simplifying multiplication. Suppose you want to multiply 8 times 105. Using the distributive property:

$$8 \cdot 105 = 8(100 + 5) = 800 + 40 = 840$$

Model:

9 • 99
Step 1 Think 99 = 100 – 1
Step 2 So 9 • 99 = 9(100 – 1)
Step 3 = 900 – 9
Step 4 Therefore, 891 is the product of 9 • 99.

Use the distributive property to perform the following multiplications.

1.305 5 • 23 = 5(20 + 3) = 100 + 15 = _____

1.306 4 • 21 = 4(20 + 1) = a. _____ + b. _____ = c. _____

1.307 7 • 15 = 7(10 + 5) = a. _____ + b. _____ = c. _____

1.308 6 • 17 = a. _____ = b. _____ = c. _____

1.309 8 • 14 = a. _____ = b. _____ = c. _____

1.310 8 • 15 = a. _____ = b. _____ = c. _____

1.311 6 • 12 = a. _____ = b. _____ = c. _____

1.312 9 • 98 = 9(100 – 2) = 900 - 18 = _____

1.313 9 • 9 = 9(10 – 1) = a. _____ = b. _____

1.314 8 • 97 = a. _____ = b. _____ = c. _____

1.315 9 • 102 = a. _____ = b. _____ = c. _____

1.316 7 • 19 = a. _____ = b. _____ = c. _____

1.317 5 • 29 = a. _____ = b. _____ = c. _____

1.318 12 • 102 = a. _____ = b. _____ = c. _____

VARIABLES

Most applications of the distributive property involve one or more variables.

Models:

Products		Sums or Differences
$6(x + 2)$	=	$6x + 12$
$x(x + 4)$	=	$x^2 + 4x$
$A(B + C)$	=	$AB + AC$
$(x + 3)5$	=	$5x + 15$
$(R + 4)R$	=	$R^2 + 4R$
$5(x - 5)$	=	$5x - 25$
$x(x - 5)$	=	$x^2 - 5x$
$A(B - C)$	=	$AB - AC$
$(A - 3)4$	=	$4A - 12$
$(N - 5)N$	=	$N^2 - 5N$

Change the following products to sums or differences.

Models: $5(x + 3) = 5 \cdot x + 5 \cdot 3 = 5x + 15$
$2m(m - 4) = 2m \cdot m - 2m \cdot 4 = 2m^2 - 8m$

1.319 $6(x + 4)$ = _____

1.320 $7(A - 6)$ = _____

1.321 $12(A - B)$ = _____

1.322 $20(A + B)$ = _____

1.323 $10(N + 3)$ = _____

1.324 $(x + 2)3$ = _____

1.325 $(x - 6)5$ = _____

1.326 $N(N - 7)$ = _____

1.327 $p(3 + p)$ = _____

1.328 $p(5 - p)$ = _____

1.329 $x(4 - x)$ = _____

1.330 $5(x^2 + 6)$ = _____

1.331 $7(x^2 + 6x)$ = _____

1.332 $12(2x + 1)$ = _____

1.333 $3(5x - 4)$ = _____

1.334 $4(x^2 + x + 1)$ = _____

1.335 $5(N^2 + 2N - 1)$ = _____

1.336 $6(A^2 - A - 4)$ = _____

1.337 $8(p^2 + 3p - 4)$ = _____

1.338 $16(4 - 2K + k^2)$ = _____

1.339 $9(y^2 + 5y + 6)$ = _____

1.340 $x(x^2 + 2x)$ = _____

1.341 $p(p^2 - 3p)$ = _____

1.342 $N(N^2 + 2N + 1)$ = _____

1.343 $R(3R^2 - 2R - 1)$ = _____ **1.344** $2x(x^2 + 3x + 5)$ = _____

1.345 $6x(2x^2 + 3x)$ = _____ **1.346** $15x(5x^2 + 6x + 3)$ = _____

1.347 $x^2(x^2 + 2x + 1)$ = _____

Change the sums or differences to products.

Models: $5x + 5 = 5 \cdot x + 5 \cdot 1 = 5(x+1)$
$x^2 - 2x = x \cdot x - x \cdot 2 = x(x - 2)$

1.348 $6x + 12$ = _____ **1.349** $7x + 14$ = _____

1.350 $8x - 16$ = _____ **1.351** $12x + 36$ = _____

1.352 $13x - 26$ = _____ **1.353** $10A - 20$ = _____

1.354 $A^2 + 5A$ = _____ **1.355** $P^2 - 10P$ = _____

1.356 $B^2 + 6B$ = _____ **1.357** $x^3 + x^2$ = _____

1.358 $6x^2 + 6y^2$ = _____ **1.359** $6A + 6B + 6C$ = _____

SIMPLIFYING EXPRESSIONS

Algebraic expressions can be simplified by combining, through addition or subtraction, as many *variable terms* or *constant terms* as possible.

In the expression $6(x + 2) + 3$, before any addition can be done, the first term, the product of $6(x + 2)$, must be rewritten as a sum: $6(x + 2) + 3 = 6x + 12 + 3$. Now we may add the two constants 12 and 3, and we get $6(x + 2) + 3 = 6x + 15$. $6x$ and 15 cannot be added; therefore, the expression $6x + 15$ is an indicated sum in its simplest form.

Model 1: Simplify, if possible, $7(A + 3) - 10$

Solution: $7(A + 3) - 10 = 7A + 21 - 10$
$= 7A + 11$

Model 2: Simplify $10 + 3(2x + 6) - 20$

Solution:
$10 + 6x + 18 - 20$
$6x + 10 + 18 - 20$
$6x + 28 - 20$
$6x + 8$

Simplify each of the following expressions. Show your work.

1.360 $7(x + 2) + 12$

_____

1.361 $8(x + 6) - 10$

_____

1.362 $13(x + 2) + 13$

_____

1.363 $10(2x + 3) - 20$

_____

1.364 $15(x + 1) + 5$

_____

1.365 $4(x + 1) - 4$

_____

1.366 $12 + 3(4 + x)$

_____

1.367 $15 + 6(x + 1)$

_____

1.368 $18(x + 1) - 9$

_____

1.369 $7(2x + 1) - 7$

_____

1.370 $4(3x + 3) - 10$

_____

1.371 $(2x + 3)5 + 6$

_____

1.372 $10 + 4(x + 1) + 5$

_____

1.373 $12 + 3(2x - 3) + 4$

_____

1.374 $18 + 5(2x - 1) + 3$

_____

1.375 $14 + 2(3x + 8) - 22$

_____

Some variable terms may be combined by using the distributive property. In the expression $3x + 4x$, the terms $3x$ and $4x$, both products, have a *common factor* x; that is, both numbers 3 and 4 multiplied by the number x.

Model 1: $3x + 4x = (3 + 4)x$
$= 7x$

Model 2: $7x + 2x = (7 + 2)x$
$= 9x$

Model 3: $7x - 2x = (7 - 2)x$
$= 5x$

The same distributive property also tells us that $6x + 4y$ cannot be combined. Why?

Terms with *like* variables (the same variables with the same respective exponents) may be combined by adding or subtracting their numerical coefficients.

Models:

$6x - 4x = (6 - 4)x = 2x$

$8y + 15y = (8 + 15)y = 23y$

$7AB - 3AB = (7 - 3)AB = 4AB$

$5N^2 + 7N^2 = (5 + 7)N^2 = 12N^2$

$8p^2 + 7p$ = (cannot be combined since the exponents are not the same)

Simplify by combining terms when possible.

1.376 $8x + 3x$

_____

1.377 $2x + x$

_____

1.378 $5x + 8x$

_____

1.379 $12x + 3x$

_____

1.380 $15x + 2x$

_____

1.381 $7x - 5x$

_____

1.382 $4x - x$

_____

1.383 $10x - 3x$

_____

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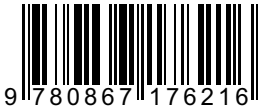

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