Are You Ready? *Introduction to Number Theory*, by M. Crawford

If you've mastered division, remainders, integer exponents and basic equations as illustrated in the problems below, then you are ready for the Art of Problem Solving book **Introduction to Number Theory**. (Answers to these problems are on the following page.)

1. **Remainders**. Find the remainder in each division problem.
   (a) $248 \div 8$
   (b) $399 \div 13$
   (c) $1333 \div 109$
2. **Integer exponents**. Evaluate the following. Express your answer as an integer or as a fraction in lowest terms.
   (a) $(-1)^6$
   (b) 3^{2+1}
   (c) 12^{-1}
   (d) $2 \cdot \left(2^3\right)^2$
3. **Equations**. Solve each of the following for x.
   (a) $4x + 7 = 23$
   (b) $5x + 10 = 54$
   (c) $3x - 4 = x + 2$
   (d) $x^2 + 2x - 4 = 0$
4. **Algebraic expressions**.
   (a) Simplify $(3x + 2) + (5x + 7)$.
   (b) Expand the product $(4n + 1)(4n + 3)$.
   (c) Factor $49 - 16x^2$.
   (d) Factor $3y^2 + 7y + 2$.
5. **Problem solving**.
   (a) The sequence $5, 7, 8, 9, 5, 7, 8, 9, \ldots$ continues to repeat with the same pattern. What is the 79th number on the list?
   (b) When the integer n is divided by 12, the remainder is 2. What is the remainder when n is divided by 6?

Don't look at the next page until you've attempted all the problems!

The answers to Are You Ready for Introduction to Number Theory are below.

1. **Remainders**.
   (a) 0
   (b) 9
   (c) 25
2. **Integer exponents**.
   (a) 1
   (b) 27
   (c) $\frac{1}{12}$
   (d) 128
3. **Equations**.
   (a) 4
   (b) $\frac{44}{5}$
   (c) 3
   (d) $-1+\sqrt{5}, -1-\sqrt{5}$
4. **Algebraic expressions**.
   (a) $8x+9$
   (b) $16n^2+16n+3$
   (c) $(7+4x)(7-4x)$
   (d) $(3y+1)(y+2)$
5. **Problem solving**.
   (a) 8
   (b) 2