



Math Fluency Intervention

Sample Lessons

Build math fluency
through Math Talks
& Number Strings



Number Strings

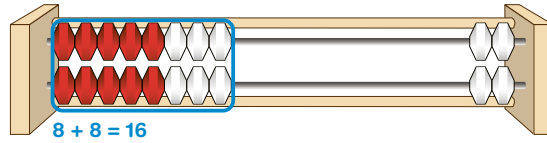
Use Doubles

17

$$8 + 8$$

Show 8 on top,
8 on the bottom

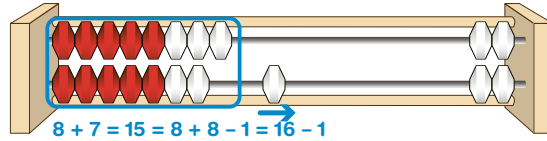
Teacher: How many beads? How do you see them? **Goal:** 16 beads; I know my doubles.



$$8 + 7$$

Show 8 on top,
7 on the bottom

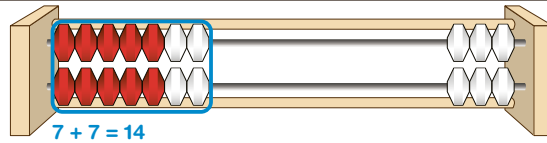
Teacher: How many beads? How do you see them? **Goal:** 15 beads; I know $8 + 8 = 16$, but that is one too many. It is 15.



$$7 + 7$$

Show 7 on top,
7 on the bottom

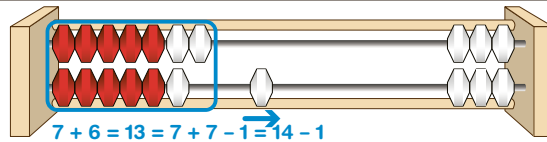
Teacher: How many beads? How do you see them? **Goal:** 14 beads; I know my doubles.



$$7 + 6$$

Show 7 on top,
6 on the bottom

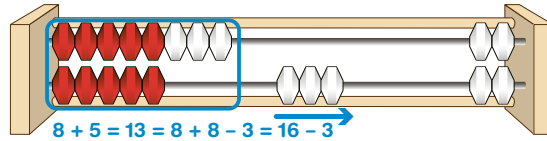
Teacher: How many beads? How do you see them? **Goal:** 13 beads; I know $7 + 7$ is 14, and minus 1 is 13.



$$8 + 5$$

Show 8 on top,
5 on the bottom

Teacher: How many beads? How do you see them? **Goal:** 13 beads; I know 8 and 8 is 16, but that is 3 too many. $16 - 3 = 13$.



Strategies Taught

- Counting On
- Counting Back
- Use the Five/Ten Structure
- Use Doubles
- Get to Ten
- Use Ten and Adjust
- Use Known Facts

Manipulatives included

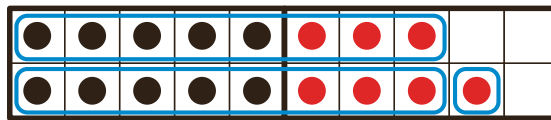
- Picture and Dot Cards
- Ten-Frames
- Double Ten-Frames
- Rekenreks
- Open Number Lines
- Equations



8 + 9

Use Doubles

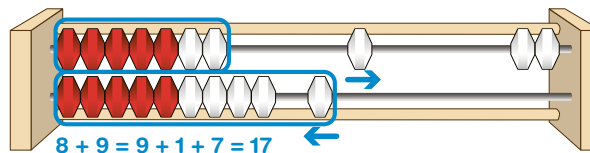
17; I know 9 is made up of 8 and 1, so I do $8 + 8$, which is 16, and 1 more is 17.



$$8 + 9 = 8 + 8 + 1 = 17$$

Get to Ten

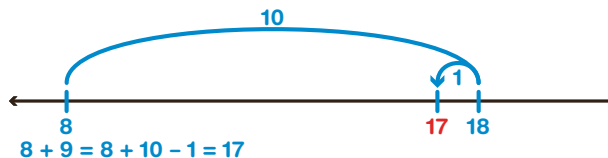
17; I take 1 from the 8 and give it to the 9. Then I do the problem $7 + 10$, which is 17.



$$8 + 9 = 9 + 1 + 7 = 17$$

Use Ten and Adjust

17; I notice that 9 is close to 10, so I think $8 + 10 = 18$. Then I subtract 1 to get to 17.



$$8 + 9 = 8 + 10 - 1 = 17$$

Use the Five/Ten Structure

17; I think about 8 as 5 and 3, and 9 as 5 and 4. So, $5 + 5 = 10$, and $3 + 4 = 7$, then $10 + 7 = 17$.

Teacher Notes

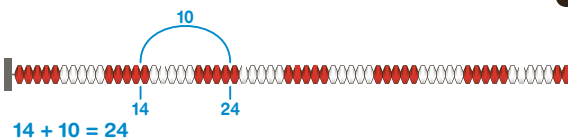
Start the Math Talk by writing $8 + 9$. Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

Facilitating Questions: 1. Can you find two strategies that are similar? How are they the same? 2. Are there any strategies that are more efficient than another? Why? 3. After observing other strategies, did you revise your thinking? How?

14 + 10

Write problem

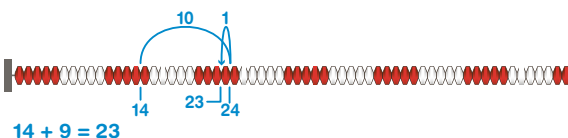
Teacher: What is 14 + 10? **Goal:** 24; I start at 14 and jump 10 to get to 24.



14 + 9

Write problem

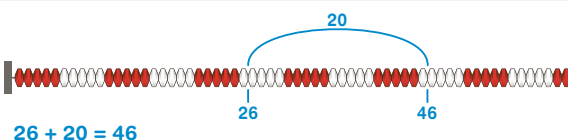
Teacher: What is 14 + 9? **Goal:** 23; I start at 14 and jump 10, which gets me to 24, but that is 1 too many, so I adjust 1 and land on 23.



26 + 20

Write problem

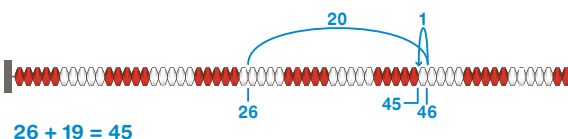
Teacher: What is 26 + 20? **Goal:** 46; I start at 26 and jump 20 to get to 46.



26 + 19

Write problem

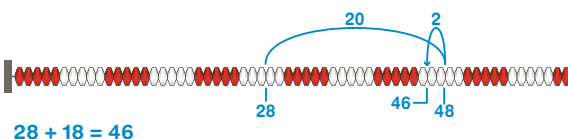
Teacher: What is 26 + 19? **Goal:** 45; I start at 26 and jump 20 to get to 46, but I only need to go 19. I adjust 1 to get 45.



28 + 18

Write problem

Teacher: What is 28 + 18? **Goal:** 46; I start at 28 and jump 20 to get to 48. Then I go back 2, which gets me to 46.



Strategies Taught

- Counting On
- Counting Back
- Use the Five/Ten Structure
- Use Doubles
- Get to Ten
- Use Ten and Adjust
- Use Known Facts

Manipulatives included

- Picture and Dot Cards
- Ten-Frames
- Double Ten-Frames
- Rekenreks
- Open Number Lines
- Equations



62 + 59

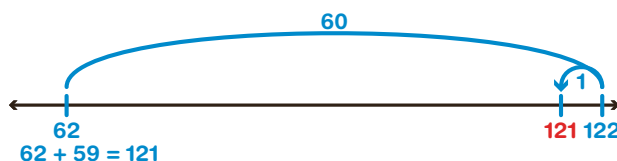
Give and Take

121; I notice that 59 is 1 away from 60. I take 1 from 62 and give it to 59 to make a new problem. I solve the problem $61 + 60$, which is 121.

$$\begin{array}{r} 62 + 59 \\ -1 \quad +1 \\ \hline 61 + 60 \end{array} \quad 62 + 59 = 121$$

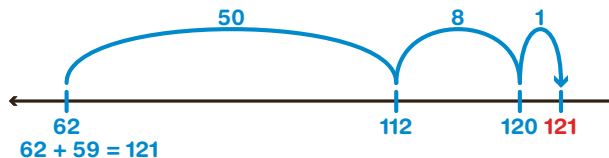
Over and Adjust

121; I think about 59 as 60. I start at 62 and jump 60 to get to 122. Then I have to adjust back 1 to get to 121.



Add a Friendly Number

121; I start at 62 and jump 50 to get to 112. Then I jump 8 to get to 120 and 1 more to get to 121.



Splitting

121; I think about 62 as $60 + 2$, and 59 as $50 + 9$. So, $60 + 50 = 110$, and $2 + 9 = 11$. Then, $110 + 11 = 121$.

Teacher Notes

Start the Math Talk by writing $62 + 59$. Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

Facilitating Questions: 1. Can you find two strategies that are similar? How are they the same? 2. Are there any strategies that are more efficient than another? Why? 3. After observing other strategies, did you revise your thinking? How?

Number Strings

Double

$$2 \times 7$$

Show a 2 x 7 array

Teacher: How many squares? How do you know? **Goal:** 14; I see 7 squares on the top row, and I just double it. So, 7 and 7 is 14.

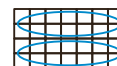


$$7 + 7 = 14 = 2 \times 7$$

$$4 \times 7$$

Show a 4 x 7 array

Teacher: How many squares? How do you know? **Goal:** 28; I know 2 rows of 7 is 14, and I just double it. So, 14 and 14 is 28.

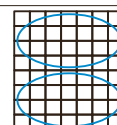


$$4 \times 7 = 28$$

$$8 \times 7$$

Show an 8 x 7 array

Teacher: How many squares? How do you know? **Goal:** 56; I know 4 rows of 7 is 28, and I just double it. So, 28 and 28 is 56.



$$8 \times 7 = 56$$

$$3 \times 7$$

Show a 3 x 7 array

Teacher: How many squares? How do you know? **Goal:** 21; I know 2 rows of 7 is 14, so I just add another 7 to get 21.

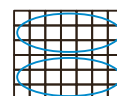


$$3 \times 7 = 21$$

$$6 \times 7$$

Show a 6 x 7 array

Teacher: How many squares? How do you know? **Goal:** 42; I know 3 rows of 7 is 21, so I just double it to get 42.



$$6 \times 7 = 42$$



Strategies Taught

- Double
- Use Partial Products
- Use Five Times
- Use Ten Times
- Double and Halve
- Multiply Up
- Partial Quotients
- Use Relationships

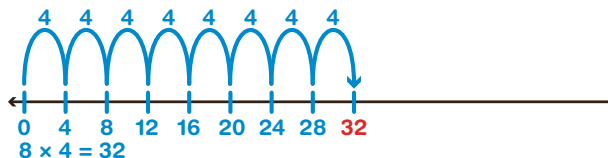
Manipulatives included

- Picture and Dot Cards
- Equations
- Open Number Lines
- Flexitable Grid Arrays
- Open Arrays
- Ratio Tables

8×4

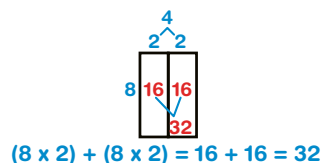
Skip Counting

32; I skip count 4, 8, 12, 16, 20, 24, 28, 32.



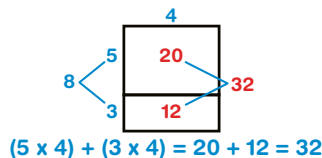
Double

32; I know 8×2 is 16, so I just double it to get 32.



Use Partial Products

32; I know 5×4 is 20 and 3×4 is 12, so 20 and 12 is 32.



Double and Halve

32; I double 8 to get 16 and halve 4 to get 2. Then I solve the problem 16×2 , which is 32.

Teacher Notes

Start the Math Talk by writing 8×4 . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

Facilitating Questions: **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than another? Why? **3.** After observing other strategies, did you revise your thinking? How?

Number Strings

Use Partial Products

40 x 46 Write problem	Teacher: What is 40×46 ? Goal: 1,840; I think about 40 as $10 + 10 + 10 + 10$. I know 46×10 is 460. I double it to get 920 and double again to get 1,840.	<p>$40 \times 46 = 1,840$</p>
4 x 46 Write problem	Teacher: What is 4×46 ? Goal: 184; I double 46 to get 92 and double 92 to get 184.	<p>$4 \times 46 = 184$</p>
44 x 46 Write problem	Teacher: What is 44×46 ? Goal: 2,024; I notice I could use the two problems from before. I think about 44 as 40 and 4. So, 40×46 and 4×46 is $1,840 + 184 = 2,024$.	<p>$44 \times 46 = (40 \times 46) + (4 \times 46) = 2,024$</p>
60 x 38 Write problem	Teacher: What is 60×38 ? Goal: 2,280; I know 38×10 is 380. I need 6 of them to get to 2,280.	<p>$60 \times 38 = 2,280$</p>
63 x 38 Write problem	Teacher: What is 63×38 ? Goal: 2,394; I think about the 63 as 60 and 3. I know 60×38 is 2,280, and 3×38 is 114. $2,280 + 114 = 2,394$.	<p>$63 \times 38 = (60 \times 38) + (3 \times 38) = 2,394$</p>



Strategies Taught

- Use Partial Products
- Double and Halve
- Factor and Group Flexibility
- Multiply Up
- Partial Quotients
- Use Relationships

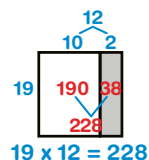
Manipulatives included

- Equations
- Flexitable Grid Arrays
- Open Arrays
- Ratio Tables

19 x 12

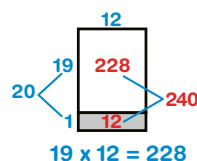
Use Partial Products

228; I think about 12 as 10 and 2. I multiply 19×10 to get 190 and 19×2 to get 38. So, $190 + 38 = 228$.



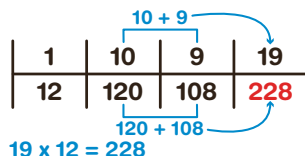
Use Partial Products

228; I think about 19 as 20. I multiply 20×12 to get 240. Then I subtract a group of 12 to get to 228.



Use Partial Products

228; I multiply 10×12 to get 120 and 9×12 to get 108. So, 120 and 108 is 228.



Double and Halve

228; I double 19 to get 38 and halve 12 to get 6. Then I double 38 to get 76 and halve 6 to get 3. Then I solve the problem 76×3 , which is 228.

Teacher Notes

Start the Math Talk by writing 19×12 . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represents the strategy.

Facilitating Questions: 1. Can you find two strategies that are similar? How are they the same? 2. Are there any strategies that are more efficient than the others? Why? 3. After observing other strategies, did you revise your thinking? How?

Number Strings

Over and Adjust

19

$6\frac{3}{8} + 2$ Write problem	Teacher: What is $6\frac{3}{8} + 2$? Goal: $8\frac{3}{8}$; I start at $6\frac{3}{8}$ and jump 2 to get to $8\frac{3}{8}$.	$6\frac{3}{8} + 2 = 8\frac{3}{8}$
$6\frac{3}{8} + 1\frac{7}{8}$ Write problem	Teacher: What is $6\frac{3}{8} + 1\frac{7}{8}$? Goal: $8\frac{2}{8}$; I notice that $1\frac{7}{8}$ is close to 2. I start at $6\frac{3}{8}$ and jump 2 to get to $8\frac{3}{8}$. Then I adjust back $\frac{1}{8}$ to get to $8\frac{2}{8}$.	$6\frac{3}{8} + 1\frac{7}{8} = 8\frac{2}{8}$
$10\frac{4}{6} + 4$ Write problem	Teacher: What is $10\frac{4}{6} + 4$? Goal: $14\frac{4}{6}$; I start at $10\frac{4}{6}$ and jump 4 to get to $14\frac{4}{6}$.	$10\frac{4}{6} + 4 = 14\frac{4}{6}$
$10\frac{4}{6} + 3\frac{5}{6}$ Write problem	Teacher: What is $10\frac{4}{6} + 3\frac{5}{6}$? Goal: $14\frac{3}{6}$; I notice that $3\frac{5}{6}$ is close to 4. I start at $10\frac{4}{6}$ and jump 4 to get to $14\frac{4}{6}$. Then I adjust back $\frac{1}{6}$ to get to $14\frac{3}{6}$.	$10\frac{4}{6} + 3\frac{5}{6} = 14\frac{3}{6}$
$14\frac{3}{10} + 6\frac{9}{10}$ Write problem	Teacher: What is $14\frac{3}{10} + 6\frac{9}{10}$? Goal: $21\frac{2}{10}$; I notice that $6\frac{9}{10}$ is close to 7. I start at $14\frac{3}{10}$ and jump 7 to get to $21\frac{3}{10}$ and adjust back $\frac{1}{10}$ to get to $21\frac{2}{10}$.	$14\frac{3}{10} + 6\frac{9}{10} = 21\frac{2}{10}$

Strategies Taught

- Splitting
- Use a Friendly Number
- Get to a Friendly Number
- Give and Take
- Over and Adjust
- Find the Distance
- Keep the Same Distance
- Use Partial Products
- Double and Halve
- Facto and Group Flexibility
- Multiply Up
- Partial Quotients
- Use Relationships

Manipulatives included

- Equations
- Open Number Lines
- Open Arrays
- Ratio Tables
- Cuisenaire® Rods
- Two-Color Counters
- Fraction Circles



$5/6 + 2/3$

Add a Friendly Number

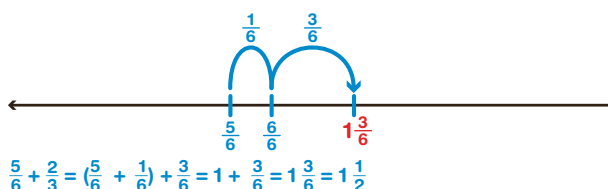
$1 \frac{1}{2}$; I think about a clock. I know $1/6$ of an hour is $10/60$, so $5/6$ is $50/60$. Then, $1/3$ of an hour is $20/60$, so $2/3$ is $40/60$. So, $50/60 + 40/60 = 90/60 = 1 \frac{30}{60} = 1 \frac{1}{2}$.

$$\frac{50}{60} + \frac{40}{60} = \frac{90}{60} = 1 \frac{30}{60} = 1 \frac{1}{2}$$

$$\frac{5}{6} + \frac{2}{3} = 1 \frac{1}{2}$$

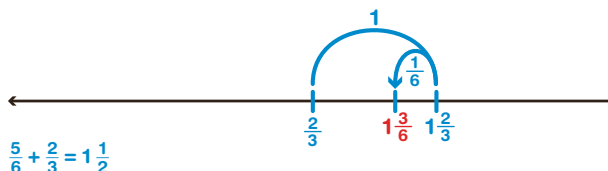
Get to a Friendly Number

$1 \frac{1}{2}$; I think about $2/3$ as $4/6$. I start at $5/6$ and jump $1/6$ to get to 1 and $3/6$ to get to $1 \frac{3}{6}$ or $1 \frac{1}{2}$.



Over and Adjust

$1 \frac{1}{2}$; I notice that $5/6$ is close to 1 . I start at $2/3$ and jump 1 to get to $1 \frac{2}{3}$, which is same as $1 \frac{4}{6}$. Then I jump back $1/6$ to get to $1 \frac{3}{6}$ or $1 \frac{1}{2}$.



Give and Take

$1 \frac{1}{2}$; I want to make an easier problem. I take $1/6$ from $2/3$ or $4/6$ and give it $5/6$. Then I solve the problem $1 + 3/6$, which is $1 \frac{3}{6}$ or $1 \frac{1}{2}$.

Teacher Notes

Start the Math Talk by writing $5/6 + 2/3$. Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

Facilitating Questions: **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than another? Why? **3.** After observing other strategies, did you revise your thinking? How?



Recommended by Teachers

"I believe Daily Math Fluency is an engaging routine that all math teachers should be implementing. I already did Number Talks, following the book by Sherry Parrish, which is very similar to Daily Math Fluency routines.

One of these programs should be followed everyday in a math classroom! I enjoyed the materials that came with the Daily Math Fluency kit, especially the flexible grid arrays. My students love these routines and is a wonderful way for students to share their math thinking in a safe environment."

—3rd grade teacher, Alabama

"There are many reasons I love hand2mind's Daily Math Fluency Kits, but one of my top reasons is the combination of Concrete-Representation-Abstract. The kits have my favorite manipulatives, but the guides included in the kits help teachers build the connections.

It's the perfect mix to help students build their fluency."

—Christina Tondevold, teacher educator & former middle school math teacher

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"Daily Math Fluency from hand2mind **helps educators easily and effectively guide math talks with students.** This allowed our teachers the framework they were looking for to be intentional about math talks in their classrooms."

—Catherine Castillo, coordinator of 21st century numeracy, Springfield (MO) Public Schools

800.445.5985
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To Learn More
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