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Introduction

Algebra is an English noun of Latin and Arabic origin. The original sense was "the surgical treatment of fractures," so in mathematics, it refers to restoring what is missing and equating like with like. Thus, our study of algebra will proceed according to this original sense of equating, or relating, numerical expressions and "restoring what is missing" by determining the numerical value of unknown quantities in relations.

The classical discipline of algebra is **the branch of mathematics in which arithmetical operations and formal manipulations are applied to abstract symbols rather than specific numbers**. You are proficient at applying arithmetical operations to numbers and evaluating numerical expressions, but algebra expands operations to include expressions containing one or more unknown quantities and expands the evaluation of expressions to include solving equations for unknown quantities. The key difference between arithmetic and algebra is the unknown quantity and the necessary relation to determine unknown quantities: **the equation**.

As a branch of mathematics, you may be tempted to relegate algebra to the science, technology, engineering, and math fields (STEM). But until the early twentieth century, mathematics was a discipline of the liberal arts! We can observe humans in history utilizing numbers to describe the world as early as around 2000 B.C. among the Babylonian and Mesopotamian civilizations. If we understand philosophy to be the study of the fundamental nature of knowledge, reality, and existence, then mathematics is an integral part of our pursuit of knowledge of the cosmos. This is precisely why the first true mathematicians were philosophers in ancient Greece! In fact, one of the earliest Greek philosophers, Pythagoras (570-490 B.C.), and his student Plato (428-347 B.C.), argued that the whole of the world is mathematical in nature, so the pursuit of true knowledge begins with a thorough understanding of mathematics. Thus, in the interest of a classical rather than vocational education, this course will pursue mastery of algebraic concepts as a complementary discipline oriented in the classical liberal arts rather than as a disjointed STEM discipline. Whether you want to be an engineer or not, algebra is an integral and formative part of your classical education.

Your study of algebra will proceed according to two basic premises:

1. **Algebra is the study of relationships.** Just as geometry focuses on spatial concepts and calculus is concerned with rates of change, algebra is a comprehensive exploration of mathematical relationships, including both equations and inequalities.

The most basic notion in algebra, in accordance with its etymology (see above), is the **equation**. An equation is a formal statement that two mathematical expressions are equal. The equation will form the foundation for your study of algebra.

2. **Algebraic concepts develop by degrees.** Thus, first-degree (linear) relations will be mastered before moving on to second-degree (quadratic) relations and higher, rational degree (rational and radical) relations — ultimately culminating in relations with variable degrees (exponential and logarithmic functions). You will master all aspects of each of these degrees before moving on to the next, and you will do so with a **consistent methodology** from beginning to end in order to pursue maximum integration and comprehension. Simply put, students should carry out the same operations and computational techniques the same way every time.

Therefore, the logical scope of your study of the discipline of algebra will cover mathematical relations of all degrees (premise 1). The logical sequence of concepts will begin with a mastery of first-degree relations and grow systematically to include increasingly more sophisticated degrees (premise 2). The general flow of each unit is based on the logical introduction of any new mathematical concept:

1. Define the new concept and describe it in detail.
2. Explore operations involving the new concept.
3. Examine relations involving the new concept and develop strategies to resolve them.

THE HISTORY OF ALGEBRA

In an effort to orient your study of algebra both in traditional Western liberal arts and philosophy and to understand how the discipline of algebra developed over time, you will familiarize yourself with the historical development of the concepts that form the foundation of algebra throughout this course.

The discipline of algebra developed as mathematicians pursued a consistent, systematic, and coherent method of relating mathematical expressions and operations. Many modern-day processes would not be possible apart from mathematicians striving to solve this problem. This pursuit ultimately arrived at the concept of the equation. The consolidation of the concept of the equation followed **three main threads** as they developed in the ancient Middle East and Greece, the Islamic era, and the European Renaissance:

1. Attempts to solve equations involving one or more unknown quantities.
2. The evolution of what qualifies as a legitimate number. Broader domains flexible enough to support the abstract structure of symbolic algebra were required. Simply put, philosophers as early as Pythagoras discovered that whole numbers were insufficient to account for the mathematical possibilities that they were observing in creation.
3. Refinement of a symbolic language suitable for devising, conveying, and solving entire categories of mathematical problems. Believe it or not, until the advent of our modern algebraic notation by René Descartes (1596-1650), all math was word problems!

Thus, you will find historical introductions at key points throughout the sequence of the course as your concept of what qualifies as a legitimate number grows, as you learn the proper symbolic language, or vocabulary, with which to engage algebra, and as you learn to solve equations of increasing degrees and with one or more unknown variables.

PRE-ALGEBRA

This course is the first of three *Memoria Algebra* courses. While "pre-algebra" is not properly a mathematical discipline, it aptly describes the purpose of the first year of algebra instruction, which is to both finish arithmetic by mastering advanced computational skills and introduce algebraic terminology, concepts, and problem-solving methodology. This course achieves the latter through mastery of first-degree relations with one variable. The content of this course is a significant conceptual leap from what your students have studied in arithmetic. Ensure that this first year of algebra instruction lays a strong and proper foundation by matching the rigor of the material with rigor in memorization of important terms and recitation, exercise completion and correction, and completion of the *Quizzes & Tests*. This course is the necessary prerequisite for *Memoria Algebra I*.

CURRICULUM COMPONENTS

The *Memoria Pre-Algebra* curriculum consists of these complementary tools:

1. *Student Text*
2. *Teacher Manual & Solutions Key*
3. *Quizzes & Tests*
4. Instructional Videos

The instructional videos are intended to serve as a supplement to the written curriculum and are not required for success in the course. The co-author of this curriculum taught this material to a class of students at Highlands Latin Cottage School in Louisville, Kentucky. The class met once weekly for ninety minutes, so some lessons received more attention than others due to the time limitations. You will find that the lesson notes in the *Teacher Manual* match the presentation of the material in the videos closely. Use the videos to reinforce lesson content as desired, especially when students struggle with certain concepts.

The primary features of the *Student Text*, besides providing problem banks with which students can work toward mastery, include clear statements of lesson objectives, introduction and review of important terms, and detailed consideration of example problems. Ensure that students master the terms and utilize the examples as the main instructional tool as they move toward completing exercises independently. The first couple exercises can be used for further demonstration as required. All exercises should be completed, checked for accuracy, and corrected for each lesson, and problematic exercises can be repeated for mastery. An answer key for the odd exercises is included in the Appendix. Ensure that students have met the lesson objective before moving on.

The *Quizzes & Tests* evaluate student progress. Often open-response questions are utilized that require students to state, in writing, their understanding of the concept under examination. This is an invaluable skill that ensures that students understand the mathematical concepts they are learning rather than becoming mere computers. Throughout this course, there are ten quizzes, one at the conclusion of each part, and three tests, one at the conclusion of each unit. Quizzes focus on the material in their respective parts, but exams are cumulative. There is also a Final Exam covering Units I-III that should be attempted after intensive review. Mastery of the material presented in this course should translate to a score of 85 or greater on the Final Exam. Reserve Form B assessments for retakes or for Final Exam review in the closing weeks of the course.

The *Teacher Manual* serves as the primary instructional resource for the course and includes an answer key for all lesson exercises. Before presenting a lesson to students, thoroughly review the material and the teacher notes for helpful techniques and emphases. A teacher or homeschool instructor with an elementary understanding of algebra will be able to convey the lesson concepts and achieve the lesson objectives by understanding and following the material included in the *Teacher Manual*. The *Solutions Key* contains complete, detailed solutions for every exercise to ensure comprehension.

A recitation schedule is located in the Appendix of the *Student Text* and *Teacher Manual*. Students should master the responses to the recitation questions for each lesson, reviewing recitation questions as necessary until they are mastered.

IS YOUR STUDENT READY FOR PRE-ALGEBRA?

The transition from arithmetic to algebra is a challenging one, so determining student readiness is key. Your student should have mastered arithmetic facts—including operations with fractions, decimals, and percentages—in order to be ready to succeed in this course. If there is any doubt, a Readiness Exam is included that you can administer and grade. The material on the exam is the equivalent to the content of an arithmetic course typically completed in sixth grade. If your student scores 90 or greater, you can be confident that he or she will succeed in this course. If your student scores between an 80 and 90, you should consider some review prior to beginning this course. If your student scores less than 80, another year of arithmetic would be best in order to ensure maximum retention of the material and to avoid exasperation due to an inability to comprehend. Algebra represents a move away from mere fact retention and toward logical reasoning, and different students will be ready for that transition at different times. Your goal should be to ensure student readiness in the interest of all involved.

UNIT I

Fundamentals

During this unit, you will review the foundational concepts that you learned in arithmetic concerning mathematical language, the numerical system, and mathematical operations. Additionally, you will be introduced to the concepts of relations, sets, and functions that form the backbone of the discipline of algebra. Therefore, you need to ensure that you have mastered the material in this unit before you proceed to Unit II.

PART A: MATHEMATICS AS A LANGUAGE

At this point in your education, you are surely aware of the importance of language. Without a shared conception of language between a communicator and listener, there can be no understanding. Thus, the purpose of Part A is to ensure that you, the student, and your textbook are communicating in a shared language. The principles that you learn in Part A will remain relevant throughout the course. Therefore, you must commit this information to memory.

Extant evidence in the form of cuneiform texts preserved in clay tablets exists from as early as 1800 B.C. showing that for centuries mathematical problems were stated and solved verbally without the use of symbols. In other words, mathematics did not operate based on a distinct language from that used in other disciplines. For instance, one of the most famous math textbooks ever produced and that remained in use for centuries, Euclid's *Elements* (c. 300 B.C.), contained no numerals! All of his proofs and theorems were written out in paragraph form. This reality limited the development of mathematical concepts for centuries.

Not until the rise of the abacists in the fourteenth century did Western society first begin to see the development of a distinct mathematical language in the form of abbreviations for unknowns. The work of Girolamo Cardano (1545) and Francois Viète (late sixteenth century) further systematized this developing language, and Pierre de Fermat and René Descartes (seventeenth century) were the first to represent polynomial equations systematically and coherently in full symbolism. The language that we use in mathematics today is the direct result of these developments that spanned millennia.

Why is mathematical language necessary? And why do you need to understand how to use it? According to Pythagoras, Plato, and all of Western philosophical history after them, the universe *is* mathematical. In fact, Plato's *Republic* set forth his vision for the ideal education, which consisted of seven disciplines, or liberal arts. Four out of the seven are mathematical disciplines: arithmetic, geometry, astronomy (the study of the motion of heavenly bodies), and music. You have little hope of understanding the world in which you live apart from learning to think and communicate mathematically.

LESSON 1

Just as language makes use of nouns, verbs, adjectives, and the like, so our mathematical language utilizes various parts of speech. The objective of this lesson is to *identify* the five parts of speech and *understand* their use.

We will consider mathematical language in five parts:

- 1. **Number symbols:** a.k.a. *numerals*, these function like "nouns" in speech.
- 2. **Operation symbols:** Indicate *action* and thus function like "verbs."
- 3. **Relation symbols:** Indicate *comparisons* and *combinations*. Think of these as "logical conjunctions" that define the relationship between mathematical expressions, or combinations of numbers and operations.
- 4. **Grouping symbols:** These function like punctuation marks that separate combinations of nouns and verbs into clauses and sentences.
- 5. **Variables:** a.k.a. *placeholder symbols*, function like "pronouns" in that they hold the place of a numeral (noun) in a mathematical sentence.

It is imperative that students possess a thorough and accurate understanding of these parts of mathematical language to correctly read and translate mathematical sentences into English or vice versa. If your students have struggled in the past with word problems, even if they are skilled in Latin and English, it is likely because of a cognitive disconnect between English and mathematical language. The creation and use of flashcards are highly recommended to learn these terms.

EXERCISE KEY

- 1. **Variable symbol:** Holds the place of a number until number is identified
- 2. **Grouping symbol:** Brackets
- 3. **Number symbol:** Three and forty-five hundredths repeating
- 4. **Operation symbol:** Add
- 5. **Relation symbol:** Is equal to

Lesson 1: Mathematical Parts of Speech

OBJECTIVE: Identify the five parts of mathematical speech and understand their use.

IMPORTANT TERMS

- 1. **Number Symbols:** More commonly called *numerals*, symbols used to represent quantities.
- 2. **Operation Symbols:** Symbols such as + (add), - (subtract), x (multiply), and ÷ (divide), used to indicate *action*.
- 3. **Relation Symbols:** Symbols such as = (is equal to), > (is greater than), and < (is less than), used to show *comparisons*, as well as *combinations* such as ≥ (is greater than or equal to), ≤ (is less than or equal to), and the negations of all of these (≠, ≠, ≠, ≠), among others.
- 4. **Grouping Symbols:** Symbols such as () (parentheses), [] (brackets), and { } (braces), used to show *groupings*.
- 5. **Variables:** Symbols such as *a, b, c* (letters of the alphabet) and □ (empty boxes), used to hold the place of a number until the number has been identified. Thus, they can vary in value.

EXAMPLE 1

Tell which of the following are relation symbols.

- a. $17\frac{1}{2}$ b. $\neq$ c. $>$ d. $\div$

Solution

- a. $17\frac{1}{2}$ is not a relation symbol. It is a number symbol meaning seventeen and one-half.
- b. $\neq$ is a relation symbol meaning "is not equal to."
- c. $>$ is a relation symbol meaning "is greater than."
- d. $\div$ is not a relation symbol. It is an operation symbol meaning "divide."

EXERCISES

Tell what part of mathematical speech each of the following is, and state in words what each means.

- | | | | | |
|----------------------|---------------|------------|-------------------|--------------|
| 1. c | 5. $=$ | 9. $-$ | 13. $\nless$ | 17. $\times$ |
| 2. $[]$ | 6. 63,231 | 10. $\{\}$ | 14. $()$ | 18. n |
| 3. $3.\overline{45}$ | 7. $\geq$ | 11. 10^4 | 15. $\frac{2}{3}$ | 19. $\neq$ |
| 4. $+$ | 8. $\sqrt{5}$ | 12. $\div$ | 16. 41.765 | 20. $\cdot$ |

- 6. **Number symbol:** Sixty-three thousand, two hundred thirty-one
- 7. **Relation symbol:** Is greater than or equal to
- 8. **Number symbol:** Approximately 2.2
- 9. **Operation symbol:** Subtract
- 10. **Grouping symbol:** Braces
- 11. **Number symbol:** 10,000
- 12. **Operation symbol:** Divide
- 13. **Relation symbol:** Is not less than
- 14. **Grouping symbol:** Parentheses
- 15. **Number symbol:** Two-thirds
- 16. **Number symbol:** Forty-one and seven hundred sixty-five thousandths
- 17. **Operation symbol:** Multiply
- 18. **Variable symbol:** Holds the place of a number until number is identified
- 19. **Relation symbol:** Is not equal to
- 20. **Operation symbol:** Multiply

Lesson 2: Mathematical Expressions

OBJECTIVE: Identify the four types of mathematical expressions and execute the appropriate actions with each.

IMPORTANT TERMS

1. **Closed Phrase:** Any mathematical expression that contains neither a relation symbol nor a variable. For example, $2(5 - 1)$ is a closed phrase.
2. **Open Phrase:** Any mathematical expression that contains a variable, but does not contain a relation symbol. For example, $6 + [3 - n]$ is an open phrase.
3. **Closed Sentence:** Any mathematical expression that contains a relation symbol, but does not contain a variable. For example, $7(4 - 2) = 13$ is a closed sentence.
4. **Open Sentence:** Any mathematical expression that contains both a relation symbol and a variable. For example, $9 + 2[n + 4] > 12$ is an open sentence.
5. **Domain:** The set of input values for which a phrase or sentence is defined.
6. **Range:** The set of output values for a phrase or sentence with a defined domain.

EXAMPLE 1

Identify the following mathematical expressions by looking for relation symbols and variables.

- a. $17 - x$ b. $16 - (2 \cdot 5) \geq 6$ c. $\frac{12+3}{2}$ d. $20 = 4(n - 1)$

Solution

- Open Phrase:** Because there is no relation symbol, this expression is just a phrase. But there is a variable, so the expression is open.
- Closed Sentence:** Because there is a relation symbol, this expression is a sentence. However, there is no variable, so the expression is closed.
- Closed Phrase:** Because there is no relation symbol, this expression is just a phrase. Neither is there a variable, so the expression is closed.
- Open Sentence:** There is a relation symbol, so the expression is a sentence. There is also a variable, so the expression is open.

EXAMPLE 2

Take the appropriate action for each of the following expressions. For open phrases, use a domain of $\{0, 1, 2\}$. For open sentences, use a replacement set of $\{4, 5, 6\}$.

- a. $17 - x$ b. $16 - (2 \cdot 5) \geq 6$ c. $\frac{12+3}{2}$ d. $20 = 4(n - 1)$

Solution

- Since this is an open phrase, the appropriate action is substitution from a domain and evaluation to a range. Using $\{0, 1, 2\}$, we substitute and evaluate:
 $17 - (0) \rightarrow 17$
 $17 - (1) \rightarrow 16$
 $17 - (2) \rightarrow 15$
 So the range is $\{17, 16, 15\}$.

As in our spoken language, mathematical parts of speech can be used to form mathematical phrases and sentences, a.k.a. expressions. The objective of this lesson is to *identify* the four types of mathematical expressions and *execute* the proper operations with each.

We will consider four types of mathematical expressions in two groups: **phrases** and **sentences**.

- Phrases do not contain relation symbols, while sentences do contain relation symbols.
- Phrases and sentences can be either open or closed.
 - » Closed indicates that a variable is not present. In other words, the solution of the phrase or sentence does not vary at all.
 - » On the other hand, an open phrase or sentence does contain at least one variable, and thus the solution of the phrase or the accuracy of the sentence depends on the numerical value assigned to the variable.

To summarize, communicate to your students the importance of categorizing mathematical expressions as either open or closed (Does it have a variable?) and as either a phrase or a sentence (Does it have a relation symbol?). As in Lesson 1, the creation and use of flashcards are highly recommended to master these terms.

To understand the examples and successfully progress through the exercises for this lesson, students need to be familiar with several additional terms:

1. **Domain:** The set of numerals that can be substituted for a variable in an open phrase.
2. **Range:** The counterpart to domain; the set of numerals that result from evaluating an open phrase for each member of the domain.
3. **Replacement Set:** Analogous to domain, but with reference to open sentences.
4. **Solution Set:** Analogous to range and the counterpart to replacement set, but with reference to open sentences. Not every member of a given replacement set will have a corresponding member in the solution set because replacement values can have false results when they are substituted into the open sentence (see Example 2).

Ensure that students master these terms and their appropriate application. To summarize, these terms only apply to open mathematical expressions because we are substituting numerals (members of a domain or replacement set) for variables. Domain and range apply to open phrases, while replacement set and solution set apply to open sentences.

Substitution and **evaluation** are complementary actions that constitute an integral and foundational skill in the discipline of algebra. Substitution refers to replacing a variable in an expression or equation with a number, and evaluation refers to simplifying an expression according to the order of operations after substitution. Your students must understand these terms and be able to apply them appropriately, so be sure to highlight these things as you work through the examples in this lesson.

EXERCISE KEY

1. Closed phrase
2. Open phrase
3. Open sentence
4. Closed sentence
5. Closed sentence
6. Open phrase
7. Open phrase
8. Closed sentence
9. Open sentence
10. Closed phrase
11. 30
12. Range = {2, 3, 4}
13. Solution set is { }
14. False
15. False
16. Range = {0, 2}
17. Range = {0, 9, 18}
18. False
19. Solution set is { }
20. 7

EXAMPLE 2 (continued)

- b. This expression is a closed sentence, so what we can do is tell if it is true or false.

$$\begin{aligned} 16 - (2 \cdot 5) &\geq 6 \\ 16 - 10 &\geq 6 \\ 6 &\geq 6 \end{aligned}$$

Since $6 = 6$, and that is one acceptable condition, the expression is true.

- c. Since this is a closed phrase, what we can do is evaluate it.

$$\frac{12+3}{2} \text{ or } \frac{15}{2} \text{ or } 7\frac{1}{2}$$

Its value is $7\frac{1}{2}$.

- d. This is an open sentence, so the appropriate action is to substitute from a replacement set and obtain a solution set of values that make the expression true. Using {4, 5, 6} we substitute and determine truth or falsehood.

$$\begin{aligned} 20 &= 4([4] - 1) \\ &= 4(3) \\ &= 12 \\ \text{FALSE} \end{aligned}$$

$$\begin{aligned} 20 &= 4([5] - 1) \\ &= 4(4) \\ &= 16 \\ \text{FALSE} \end{aligned}$$

$$\begin{aligned} 20 &= 4([6] - 1) \\ &= 4(5) \\ &= 20 \\ \text{TRUE} \end{aligned}$$

So our solution set is {6}.

EXERCISES

Tell whether each of the following expressions is an open phrase, closed phrase, open sentence, or closed sentence.

1. $6 + 8(3)$
2. $m + 2$
3. $5 - z = 2$
4. $8 - 2^3 = 4 - 2$
5. $\frac{12+8}{5} > 2 \cdot 2$
6. $w^2 - w$
7. $9y$
8. $\frac{15+18}{3} \neq 5 + 6$
9. $3k + 11 < 17$
10. $2 \cdot 4 - 1$

In the following exercises, take the appropriate mathematical action with each expression. For open phrases, use a domain of {0, 1, 2}. For open sentences, use a replacement set of {4, 5, 6}.

11. $6 + 8(3)$
12. $m + 2$
13. $5 - z = 2$
14. $8 - 2^3 = 4 - 2$
15. $\frac{12+8}{5} > 2 \cdot 2$
16. $w^2 - w$
17. $9y$
18. $\frac{15+18}{3} \neq 5 + 6$
19. $3k + 11 < 17$
20. $2 \cdot 4 - 1$

UNIT III

Introduction to the Cartesian Coordinate System

Solving equations results in solution sets, and as you further pursue the taming of the unknown in your study of algebra, you need to know how to state the solution set for first-degree relations with two variables. You can do so both algebraically – using set notation, with which you are already familiar – and graphically – using the Cartesian coordinate system, or the two-dimensional xy -plane, as opposed to the one-dimensional number line.

As you will shortly see, the solution set for a single first-degree relation with two variables always graphs as a straight line (linear). Thus, you will define slope and intercepts as the two most important and helpful characteristics of such graphical representations. It is important for you to seek to marry algebraic expression and graphical representation in your understanding as different depictions of the same mathematical reality. The Cartesian coordinate system is helpful to the extent that you recognize its purpose: visual, geometric depiction of an algebraic reality. In short, you can express the solution set for a linear relation in either set notation or as the graph of a line in the Cartesian plane. The (x, y) coordinates of every point that makes up the line (and there are an infinite number of points!) represent a solution to the relation. For such infinite solution sets, a graph is a powerful tool of communication.

The beauty of both of these systems is that they enable clear, concise, and powerful expression of otherwise rather abstract concepts.

LESSON 51

The Cartesian coordinate system is the means of graphically depicting the solution set for relations in two variables. Thus, obtaining a strong foundation in these terms and concepts and obtaining the ability to graph ordered pairs in the plane with ease are imperative.

Work with your students on mastering the ability to label the parts of the Cartesian plane, including the quadrants, and to label the coordinates in an ordered pair. In the same way that they pursued and achieved number sense in arithmetic, so they must pursue and achieve a sense with reference to the Cartesian coordinate system.

A helpful way to relate quadrant numbers to ordered pairs is to show that all ordered pairs in quadrant I have positive x - and y -coordinates, i.e. $(+, +)$. In quadrant II, all ordered pairs are $(-, +)$. All the ordered pairs in quadrant III are $(-, -)$, and those in quadrant IV are $(+, -)$. Mastering these relationships gives students a quick method for a reasonableness check regarding the accuracy of their graphs and their reading of ordered pair coordinates from graphs.

Continue to teach students that in order to graph an ordered pair, they must begin at the origin, move along the x -axis the same number of units as the x -coordinate in the proper direction (left for negative and right for positive), and then move from that point up (for positive) or down (for negative) the same number of units as the y -coordinate. This process is best taught via visual demonstration. Remember, consistency in methodology leads to mastery!

Lesson 51: Graphing Terminology

OBJECTIVE: Understand, gain familiarity with, and master the terms used to describe the Cartesian coordinate system.

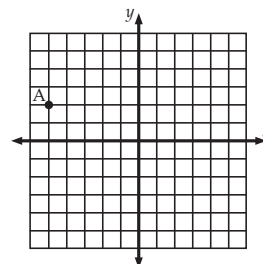
IMPORTANT TERMS

1. **Rectangular Coordinate System:** Also called the Cartesian coordinate system (named for its developer, René Descartes, a seventeenth-century French scientist and philosopher). It is a framework within which solutions for first-degree relations using two variables can be shown.
2. **Coordinate Axes:** The two number lines placed at right angles and intersecting at zero, which are used to name all of the points in a plane.
3. **Horizontal Axis:** Often called the x -axis, the horizontal number line in a rectangular coordinate system, always associated with the first variable in an ordered pair.
4. **Vertical Axis:** Often called the y -axis, the vertical number line in a rectangular coordinate system, always associated with the second variable in an ordered pair.
5. **Origin:** The point $(0, 0)$ where the coordinate axes intersect and where the counting begins to name all other points in the plane.
6. **Ordered Pair:** Often called the coordinates of a point, it is a term used to describe two numbers that locate and identify a point in the plane. They are given in a particular order and are generally grouped by using parentheses.
7. **Graph of an Ordered Pair:** The point in the plane that is located by, and identified with, a particular ordered pair.
8. **Abscissa:** From a Latin word meaning "to cut," this term refers to the first number in an ordered pair, the number that tells us where to "cut" the horizontal axis as we begin locating a point.
9. **Ordinate:** From a Latin word meaning "ordered," this term refers to the second number in an ordered pair, the number that we use, in an "ordered" way, after we use the abscissa.
10. **Quadrant:** Any one of the four sections of the plane created by the coordinate axes, they are numbered for convenience beginning at the upper right (quadrant I) and moving in a counterclockwise direction (quadrants II, III, IV).

EXAMPLE 1

Complete the statements about the point A indicated in the following rectangular coordinate system.

- a. The ordered pair is _____.
- b. The abscissa is _____.
- c. The ordinate is _____.
- d. The point is in quadrant _____.
- e. The point is _____ units (left or right) from the origin.
- f. The point is _____ units (up or down) from the origin.



EXAMPLE 1 (continued)**Solution**

- a. The point A is 5 units to the left on the horizontal axis, and left is negative. It is also 2 units up on the vertical axis, and up is positive.
So the ordered pair is $(-5, 2)$, since the horizontal axis is always considered first.
- b. The abscissa is the first number in an ordered pair. In this case, the abscissa is -5 .
- c. The ordinate is the second number in an ordered pair. In this case, the ordinate is 2 .
- d. We know the upper right quadrant is quadrant I. Moving counterclockwise, the point is in quadrant II.
- e. From the explanation above, we know the point is 5 units to the left of the origin on the horizontal axis.
- f. From the explanation above, we know the point is 2 units up from the origin on the vertical axis.

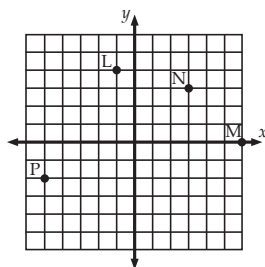
EXAMPLE 2

On the rectangular coordinate system given, plot the point indicated by each ordered pair, and label it appropriately.

- a. $L = (-1, 4)$ b. $M = (6, 0)$ c. $N = (3, 3)$ d. $P = (-5, -2)$

Solution

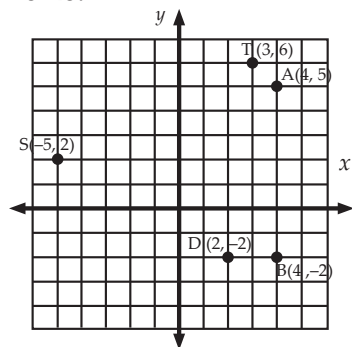
- a. The point L is 1 unit to the left of the origin and 4 units up from the origin.
- b. The point M is 6 units to the right of the origin and 0 units up or down from the origin. It is on the horizontal axis.
- c. The point N is 3 units to the right of the origin and 3 units up from the origin.
- d. The point P is 5 units to the left of the origin and 2 units down from the origin.



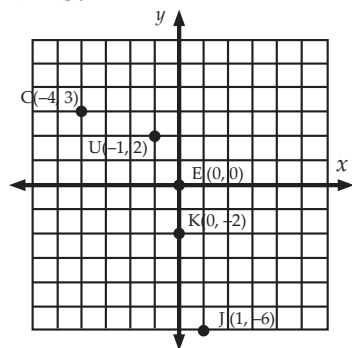
EXERCISE KEY

1. (3, 5)
2. 0
3. IV or 4
4. (6, -3)
5. 6
6. 0
7. 3
8. 3
9. -3
10. Q
11. (-4, 0)
12. 3
13. 5
14. I or 1
15. -4

16-20.

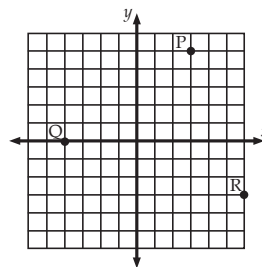


21-25.



EXERCISES

Complete the statements about the points indicated on the following rectangular coordinate system.



1. The ordered pair for point P is _____.
2. The ordinate for point Q is _____.
3. Point R is in quadrant _____.
4. The ordered pair for point R is _____.
5. The abscissa for point R is _____.
6. Point Q is _____ units up from the origin.
7. The abscissa for point P is _____.
8. Point P is _____ units to the right of the origin.
9. The ordinate for point R is _____.
10. Point _____ is on the horizontal axis.
11. The ordered pair for point Q is _____.
12. Point R is _____ units down from the origin.
13. The ordinate for point P is _____.
14. Point P is in quadrant _____.
15. The abscissa for point Q is _____.

On the same rectangular coordinate system, plot the point indicated by each ordered pair, and label it appropriately.

16. S is (-5, 2)
17. A is (4, 5)
18. D is (2, -2)
19. T is (3, 6)
20. B is (4, -2)

On the same rectangular coordinate system, plot the point indicated by each ordered pair, and label it appropriately.

21. U is (-1, 2)
22. E is (0, 0)
23. J is (1, -6)
24. C is (-4, 3)
25. K is (0, -2)