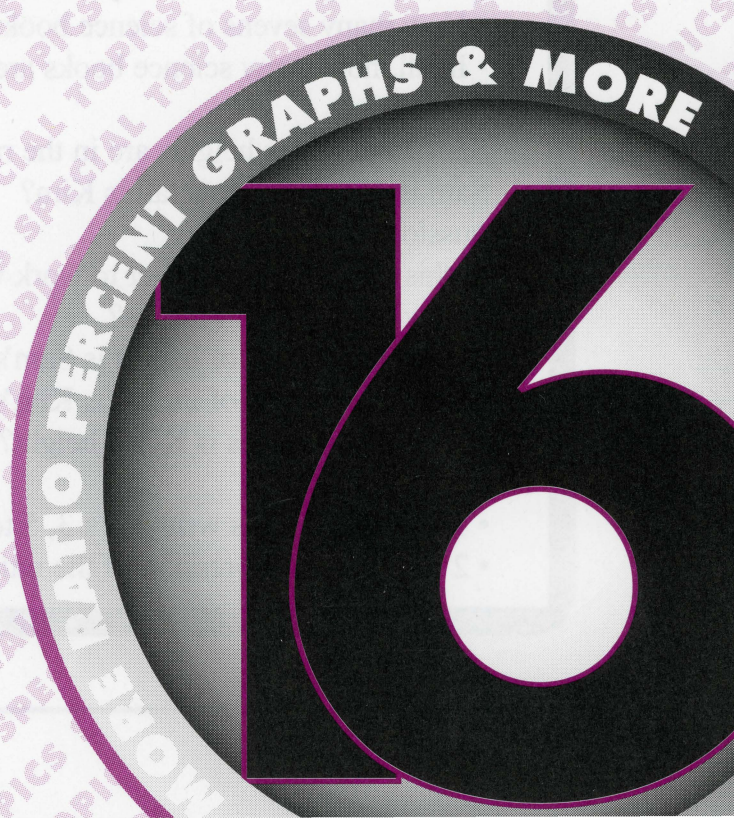


LEVEL SIXTEEN

SPECIAL TOPICS

Developmental Mathematics



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1

RATIO

Example 1.

In a school, the boys-to-girls ratio is 3 to 2.

Discussion.

It means that for every 3 boys, there are 2 girls.

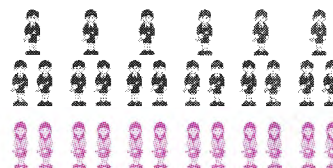
If there are 50 threes of boys, there are 50 twos of girls.

If there are 69 threes of boys, there are 69 twos of girls.

If there are 127 threes of boys, there are ____ twos of girls.

If there are 369 threes of boys, there are ____ twos of girls.

What does it mean?



Example 2.

Jack has purple and black marbles in the ratio 3 to 4.

The purple marbles are pictured in box A.

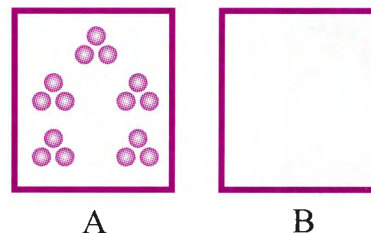
Draw a picture for the black marbles in box B.

Discussion:

3 to 4 here means that for every 3 purple marbles, there are 4 black marbles.

How many threes of purple marbles are there? _____

How many fours of black marbles are there? _____



Example 3.

In the library, the history and science books are in the ratio 4 to 7.

There are 280 history books. How many science books are there?

Discussion:

4 to 7 here means that for every 4 history books, there are 7 science books.

• How many fours of history books are there? _____

• How many sevens of science books are there? _____

• In all, how many science books are there? _____

Example 4.

Sam's and Kim's heights are in the ratio 5 to 6.

Sam is 4'2" tall. How tall is Kim?

Discussion:

In this situation, it is easier to work with inches.

• How tall is Sam in inches? _____

• How many fives of inches is Sam's height? _____

• How many sixes of inches is Kim's height? _____

• How many inches is Kim's height? _____

Notes:

• The ratio 2 to 3 is written 2 : 3, 5 to 6 is written 5 : 6, 8 to 13 is written 8 : 13, ...etc.

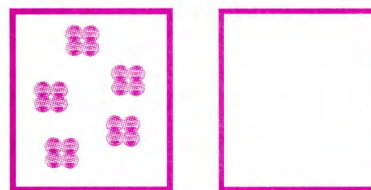
• 2 : 3 : 7 is an expression of the ratios 2 : 3, 3 : 7, and 2 : 7.

APPLICATIONS

1. The ratio of Dick's money to Mary's is 4 to 3.

Dick's money is pictured in box A.

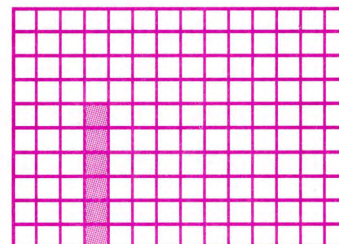
Draw Mary's money in box B.



2. The ratio of Bob's height to Ann's is 3 : 4.

On the graph paper to the right, Bob's height is represented by line X.

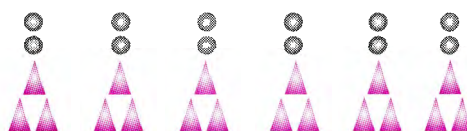
Draw a line that represents Ann's height.



3. Study the picture to the right.

a. What is the circles-to-triangles ratio? _____

b. Explain.



4. On a farm, there are 20 cows, 48 sheep, and 18 horses.

a. What is the cows-to-sheep ratio? _____

Why? _____

b. What is the cows-to-horses ratio? _____

Why? _____

5. The ratio of Jill's dolls to Kathy's is 2 to 3.

Jill has 12 dolls.

How many dolls does Kathy have?

6. On an orchard, there are golden and red apple trees in the ratio 8 : 5.

There are 480 red apple trees.

How many golden apple trees are there?

Example 5.

In a school, the boys-to-girls ratio is 3 : 2.

The student body is 450 students. How many are boys and how many girls?

Discussion:

3 : 2 here means that for every 3 boys there are 2 girls.

It also means that all students can be made into groups with 3 boys and 2 girls (5 in all) in each group.

- How many fives are the students? _____
- How many threes are the boys? _____
- How many twos are the girls? _____
- How many boys are there? _____
- How many girls are there? _____

Example 6.

John has American and foreign stamps in the ratio 6 : 5.

Altogether he has 968 stamps. How many are American, and how many are foreign?

Discussion:

6 : 5 here means that for every 6 American stamps John has, he has 5 foreign stamps.

It also means that John can make all his stamps into sets with 6 American and 5 foreign stamps (11 stamps altogether) in each set.

- In all, how many sets can he make? _____
- How many 6's of American stamps does he have? _____
- How many 5's of foreign stamps does he have? _____
- How many American stamps does he have? _____
- How many foreign stamps does he have? _____

Example 7.

You want to divide \$600 between Tom, Jill, and Sally in the ratios 3 : 4 : 5.

How much will each get?

Discussion:

3 : 4 : 5 here means that for every \$3 Tom gets, Jill gets \$4, and Sally gets \$5.

It also means that out of every \$12, Tom gets \$3, Jill gets \$4, and Sally gets \$5.

- How many 12's are in 600? _____
- How many 3's of dollars does Tom get? _____
- How many 4's of dollars does Jill get? _____
- How many 5's of dollars does Sally get? _____
- How much money does Tom get? _____
- How much money does Jill get? _____
- How much money does Sally get? _____

APPLICATIONS

1. You want to divide \$800 between Dan and Jim in the ratio 9 : 7.
How much does each one get?

2. In a school district there are 3,000 students.
Given that the male-to-female ratio is 11 : 9, find the number of male and female students.

Given that the ratio of students in elementary, junior high, and high is 6 : 5 : 4, find the number of students in each type of school.

3. You have 400 coins (quarters, dimes, and nickels) in the ratio 2 : 5 : 3.
How much money do you have?

4. The sum of two numbers A and B is 540.
Given that A : B is 3 : 7, find each of the two numbers.

5. The sum of three numbers X, Y, and Z is 700.
Given that $X : Y : Z = 2 : 7 : 5$, find each number.

6. The sum of the numbers E, F, G, and H is 369.
Given that $E : F : G : H = 23 : 39 : 21 : 17$, find each number.

2

ANOTHER MEANING FOR RATIO

Example 1.

In box A, there are 5 sets, 2 circles each.

In box B, there are 5 sets, 3 squares each.

- For every 2 circles, there are 3 squares.

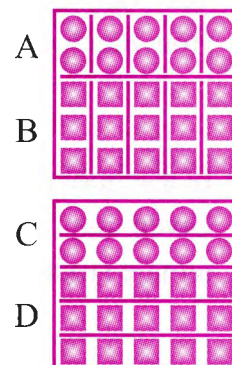
The circles-to-squares ratio is 2 to 3.

- You can make the circles into 2 sets, 5 circles each, as shown in box C.

You can make the squares into 3 sets, 5 squares each, as shown in box D.

- The ratio 2 to 3 now means that if you divide the circles equally into 2 sets, the squares will be 3 sets equivalent to the sets of circles.
- You may use fractions to describe how the number of circles and the number of squares are related.

- $\frac{1}{2}$ the number of circles is equal to $\frac{1}{3}$ the number of squares.
- The number of circles is $\frac{2}{3}$ the number of squares.
- The number of squares is $\frac{3}{2}$ the number of circles.
- The number of circles is $\frac{2}{5}$ the total number of circles and squares.
- The number of squares is $\frac{3}{5}$ the total number of circles and squares.



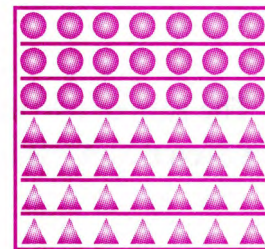
Example 2.

Study the picture to the right.

There are 3 sets of circles, 7 circles each.

There are 4 sets of triangles, 7 triangles each.

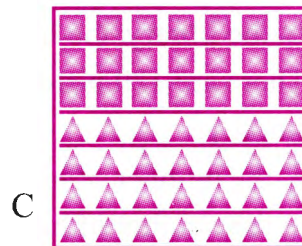
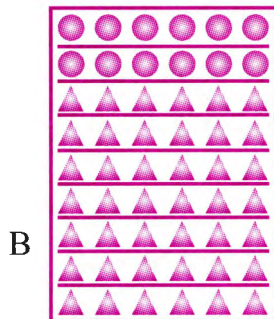
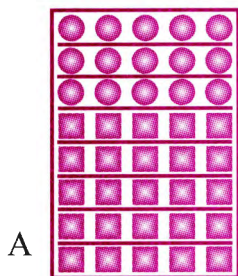
- The sets of circles and the sets of triangles are equivalent.
So, the circles-to-triangles ratio is 3 to 4.
- The ratio 3 to 4 here means that if you divide the circles equally into 3 sets, the triangles will be 4 sets equivalent to the sets of circles.



Using fractions, you can make the following statements:

- $\frac{1}{3}$ the number of circles is equal to $\frac{1}{4}$ the number of triangles.
- The number of circles is $\frac{3}{4}$ the number of triangles.
- The number of triangles is $\frac{4}{3}$ the number of circles.
- The number of circles is $\frac{3}{7}$ the total number of circles and triangles.
- The number of triangles is $\frac{4}{7}$ the total number of circles and triangles.

APPLICATIONS



1. Study picture A.

What is the circles-to-squares ratio? _____

2. Study picture A. Write the missing fractions:

- $\frac{1}{3}$ the number of circles = _____ the number of squares.
- The number of circles = _____ the total number of squares and circles.
- The number of squares = _____ the total number of squares and circles.

3. Study picture B.

What is the circles-to-triangles ratio? _____

4. Study picture B. Write the missing fractions:

- The number of triangles = _____ the number of circles.
- The number of triangles = _____ the total number of triangles and circles.
- The number of circles = _____ the total number of triangles and circles.

5. Study picture C.

What is the squares-to-triangles ratio? _____

6. Study picture C. Write the missing fractions:

- $\frac{1}{3}$ the number of squares = _____ the number of triangles.
- The number of squares = _____ the total number of squares and triangles.
- The number of triangles = _____ the total number of squares and triangles.

7. In a village, men, women and children are in the ratios 6 : 5 : 2.

- What fraction of the adults is men? _____
- What fraction of the adults is women? _____
- What fraction of the population is men? _____
- What fraction of the population is women? _____
- What fraction of the population is children? _____
- What fraction of the number of men is the number of children? _____

8. Two numbers X and Y are in the ratio 5 : 7.

- What fraction of X is Y? _____
- What fraction of the sum is X? _____
- What fraction of the sum is Y? _____

3

USING FRACTIONS

Example 1.

In the library, the history-to-science books ratio is 3 : 7.

There are 120 history books. How many science books are there?

Solution 1:

- $\frac{1}{3}$ the number of history books = $\frac{1}{7}$ the number of science books.
- $\frac{1}{3}$ the number of history books = $\frac{1}{3} \times 120 = 40$
- $\frac{1}{7}$ the number of science books = 40
- The number of science books = $7 \times 40 = 280$

Solution 2:

The number of science books = $\frac{7}{3}$ the number of history books.

So, the number of science books = $\frac{7}{3} \times 120 = 280$

Example 2.

Roads A and B are in the ratio 2 : 3.

A _____

Road B is 24 miles long. How long is Road A?

B _____

Solution 1:

- $\frac{1}{2}$ the length of Road A = $\frac{1}{3}$ the length Road B.
- How long is $\frac{1}{3}$ of Road B? _____
- How long is $\frac{1}{2}$ of Road A? _____
- How long is Road A? _____

Solution 2:

The length of Road A = $\frac{2}{3}$ the length of road B.

- The length of Road B is 24 miles.
- How long is Road A? _____

Example 3.

John, Jack, and Sue have farming land in the ratios 3 : 2 : 4.

Jack owns 50 acres. How many acres does John own,
and how many acres does Sue own?

Solution:

- The number of acres John owns = $\frac{3}{2}$ the number of acres Jack owns.
The number of acres John owns = _____
- The number of acres Sue owns = $\frac{4}{2}$ the number of acres Jack owns.
The number of acres Sue owns = _____

APPLICATIONS

1. On an airplane, there are 300 tourist class seats, and a number of first class seats in the ratio 25 : 2.
How many first class seats are there?

2. In a school, the teacher-to-student ratio is 3 : 50.
There are 51 teachers. How many students are there?

3. You want to divide \$750 between Bill, Jack, and Nancy in the ratios 3 : 5 : 7.
Find the amount each person gets.

4. 15 and a number A are in the ratio 3 : 4.
 - a. What fraction of 15 is A? _____
 - b. What number is A? _____
5. Two numbers X and Y whose sum is 30 are in the ratio 3 : 7.
 - a. What fraction of the sum is X? _____
 - b. What number is X? _____
 - c. What fraction of the sum is Y? _____
 - d. What number is Y? _____
6. Two numbers A and B are in the ratio 4 : 5.
 - a. What fraction of (A + B) is A? _____
 - b. What fraction of (A + B) is B? _____

Given $A + B = 18$,

 - c. What number is A? _____
 - d. What number is B? _____
7. Two numbers, M and N are in the ratio 7 : 9.
Given that $M + N = 24$, find the product of M and N.

THE RATIO AS A FRACTION IN SIMPLEST FORM

Example 1.

Dan has 30 marbles, and Helen has 40 marbles.

What is the ratio of Dan's marbles to Helen's?

Discussion:

- You may make all the marbles into equivalent sets, 2 marbles each.
Dan will have 15 sets, and Helen 20 sets.
Dan's marbles to Helen's is 15 : 20. $30 : 40 = 15 : 20$
- You may make all the marbles into equivalent sets, 5 marbles each.
Dan will have 6 sets, and Helen 8 sets.
Dan's marbles to Helen's is 6 : 8 $30 : 40 = 6 : 8$
- You may make all the marbles into equivalent sets, 10 marbles each.
Dan will have 3 sets, and Helen 4 sets.
Dan's marbles to Helen's is 3 : 4 $30 : 40 = 3 : 4$
The ratio 30 : 40 may be simplified into 15 : 20, 6 : 8, or 3 : 4.
When the ratio 30 : 40 is written as 3 : 4, it is in simplest form.
Using fractions, you may write $30 : 40 = \frac{30}{40} = \frac{15}{20} = \frac{6}{8} = \frac{3}{4}$

Example 2.

Bill's height is 5' and Dick's is 4' 6".

Compare Bill's height to Dick's.

Discussion:

The two quantities have to be in the same unit.

- You may use inches:
Bill's height in inches = $5 \times 12 = 60$
Dick's height in inches = $(4 \times 12) + 6 = 54$
Bill's height to Dick's = $60 : 54 = 10 : 9$ or $\frac{60}{54} = \frac{10}{9}$
- You may use feet:
Bill's height in feet = 5
Dick's height in feet = $4 \frac{6}{12} = 4 \frac{1}{2}$
Bill's height to Dick's is $5 : 4 \frac{1}{2} = \frac{10}{2} : \frac{9}{2} = \frac{10}{9}$
- You may use yards:
Dick's height in yards = $4 \frac{1}{2} \div 3 = 1 \frac{1}{2}$
Bill's height in yards = $5 \div 3 = 1 \frac{2}{3}$
Bill's height to Dick's is $1 \frac{2}{3} : 1 \frac{1}{2} = \frac{5}{3} : \frac{3}{2} = \frac{10}{6} : \frac{9}{6} = \frac{10}{9}$

Notes:

- The numbers that form a ratio are called the *terms of the ratio*.
- When a ratio is in simplest form, it is in lowest terms.

EXERCISES

Express each ratio in the simplest form:

a. $35 : 15 =$

b. $66 : 44 =$

c. $30 : 50 =$

d. $60 : 80 =$

e. $250 : 350 =$

f. $58 : 64 =$

g. $2 : \frac{1}{3} =$

h. $\frac{2}{5} : 3 =$

i. $\frac{2}{3} : \frac{3}{4} =$

j. $\frac{1}{2} : \frac{1}{3} =$

k. $7\frac{1}{2} : 8\frac{1}{3} =$

l. $3.6 : 5.58 =$

m. $1.05 : 7 =$

n. $6.4 : 16 =$

APPLICATIONS

1. There are 1200 students in a school.

a. There are 15 mathematics teachers.

Find the teacher-to-student ratio. _____

b. 8 buses serve the school.

Find the student-to-bus ratio. _____

2. 1,225 persons live in a village. 600 are males.

a. Find the male-to-all people ratio. _____

b. Find the male-to-female ratio. _____

3. Tammy keeps record of her earnings:

a. Compare her earnings in January to her earnings in February. _____

b. Compare her earnings in January and February to her earnings in March and April. _____

January	\$75
February	\$60
March	\$80
April	\$70

4. Carl has 25 dimes, and Frank has 40 nickels.

Compare the two amounts of money. _____

5. Two pieces of wood are 30 yds. and 60 ft. long.

Compare the two lengths. _____

6. Two pieces of meat weigh $4\frac{1}{2}$ lb. and $3\frac{3}{5}$ lb.

Compare the two weights. _____