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Introduction

Algebra is an English noun of Latin and Arabic origin. The original sense was "the surgical treatment of fractures," so in mathematics, it refers to restoring what is missing and equating like with like. Thus, our study of algebra will proceed according to this original sense of equating, or relating, numerical expressions and "restoring what is missing" by determining the numerical value of unknown quantities in relations.

The classical discipline of algebra is **the branch of mathematics in which arithmetical operations and formal manipulations are applied to abstract symbols rather than specific numbers**. You are proficient at applying arithmetical operations to numbers and evaluating numerical expressions, but algebra expands operations to include expressions containing one or more unknown quantities and expands the evaluation of expressions to include solving equations for unknown quantities. The key difference between arithmetic and algebra is the unknown quantity and the necessary relation to determine unknown quantities: **the equation**.

As a branch of mathematics, you may be tempted to relegate algebra to the science, technology, engineering, and math fields (STEM). But until the early twentieth century, mathematics was a discipline of the liberal arts! We can observe humans in history utilizing numbers to describe the world as early as around 2000 B.C. among the Babylonian and Mesopotamian civilizations. If we understand philosophy to be the study of the fundamental nature of knowledge, reality, and existence, then mathematics is an integral part of our pursuit of knowledge of the cosmos. This is precisely why the first true mathematicians were philosophers in ancient Greece! In fact, one of the earliest Greek philosophers, Pythagoras (570-490 B.C.), and his student Plato (428-347 B.C.), argued that the whole of the world is mathematical in nature, so the pursuit of true knowledge begins with a thorough understanding of mathematics. Thus, in the interest of a classical rather than vocational education, this course will pursue mastery of algebraic concepts as a complementary discipline oriented in the classical liberal arts rather than as a disjointed STEM discipline. Whether you want to be an engineer or not, algebra is an integral and formative part of your classical education.

Your study of algebra will proceed according to two basic premises:

1. **Algebra is the study of relationships.** Just as geometry focuses on spatial concepts and calculus is concerned with rates of change, algebra is a comprehensive exploration of mathematical relationships, including both equations and inequalities.

The most basic notion in algebra, in accordance with its etymology (see above), is the **equation**. An equation is a formal statement that two mathematical expressions are equal. The equation will form the foundation for your study of algebra.

2. **Algebraic concepts develop by degrees.** Thus, first-degree (linear) relations will be mastered before moving on to second-degree (quadratic) relations and higher, rational degree (rational and radical) relations – ultimately culminating in relations with variable degrees (exponential and logarithmic functions). You will master all aspects of each of these degrees before moving on to the next, and you will do so with a **consistent methodology** from beginning to end in order to pursue maximum integration and comprehension. Simply put, students should carry out the same operations and computational techniques the same way every time.

Therefore, the logical scope of your study of the discipline of algebra will cover mathematical relations of all degrees (premise 1). The logical sequence of concepts will begin with a mastery of first-degree relations and grow systematically to include increasingly more sophisticated degrees (premise 2). The general flow of each unit is based on the logical introduction of any new mathematical concept:

1. Define the new concept and describe it in detail.
2. Explore operations involving the new concept.
3. Examine relations involving the new concept and develop strategies to resolve them.

THE HISTORY OF ALGEBRA

In an effort to orient your study of algebra both in traditional Western liberal arts and philosophy and to understand how the discipline of algebra developed over time, you will familiarize yourself with the historical development of the concepts that form the foundation of algebra throughout this course.

The discipline of algebra developed as mathematicians pursued a consistent, systematic, and coherent method of relating mathematical expressions and operations. Many modern-day processes would not be possible apart from mathematicians striving to solve this problem. This pursuit ultimately arrived at the concept of the equation. The consolidation of the concept of the equation followed **three main threads** as they developed in the ancient Middle East and Greece, the Islamic era, and the European Renaissance:

1. Attempts to solve equations involving one or more unknown quantities.
2. The evolution of what qualifies as a legitimate number. Broader domains flexible enough to support the abstract structure of symbolic algebra were required. Simply put, philosophers as early as Pythagoras discovered that whole numbers were insufficient to account for the mathematical possibilities that they were observing in creation.
3. Refinement of a symbolic language suitable for devising, conveying, and solving entire categories of mathematical problems. Believe it or not, until the advent of our modern algebraic notation by René Descartes (1596-1650), all math was word problems!

Thus, you will find historical introductions at key points throughout the sequence of the course as your concept of what qualifies as a legitimate number grows, as you learn the proper symbolic language, or vocabulary, with which to engage algebra, and as you learn to solve equations of increasing degrees and with one or more unknown variables.

ALGEBRA I

This course is the second of three *Memoria Algebra* courses. If you have completed *Memoria Pre-Algebra*, you are well-prepared to succeed in this course. If you completed a different pre-algebra course, you can still pick up with *Memoria Algebra I* if you learned first-degree equations and inequalities with one variable including solving and graphing compound inequalities and absolute value relations and using single-variable equations to solve word problems. A sound foundation in these topics, as well as in algebraic terminology and symbolism, is necessary as *Memoria Algebra I* is a fast-paced and challenging course. You will complete your mastery of first-degree relations by learning solving and graphing skills for two or more variables, and then you will transition to higher-degree relations involving polynomial operations and factoring, algebraic fractions and rational functions, and radical expressions and equations. The course culminates in the broadening of your conception of a legitimate number by introducing the imaginary number and the set of complex numbers. You will need to match the rigor of the course content with your own rigor and diligence in memorization and recitation, exercise completion and correction, and successful completion of the assessments. This course is the necessary prerequisite for *Memoria Algebra II*.

CURRICULUM COMPONENTS

The *Memoria Algebra I* curriculum consists of these complementary tools:

1. *Student Text*
2. *Teacher Manual & Solutions Key*
3. *Quizzes & Tests*
4. Instructional Videos

The instructional videos are intended to serve as a supplement to the written curriculum and are not required for success in the course. The co-author of this curriculum taught this material to a class of students at Highlands Latin Cottage School in Louisville, Kentucky. The class met once weekly for ninety

minutes, so some lessons received more attention than others due to the time limitations. You will find that the lesson notes in the *Teacher Manual* match the presentation of the material in the videos closely. Use the videos to reinforce lesson content as desired, especially when students struggle with certain concepts.

The primary features of the *Student Text*, besides providing problem banks with which students can work toward mastery, include clear statements of lesson objectives, introduction and review of important terms, and detailed consideration of example problems. Ensure that students master the terms and utilize the examples as the main instructional tool as they move toward completing exercises independently. The first couple exercises can be used for further demonstration as required. All exercises should be completed, checked for accuracy, and corrected for each lesson, and problematic exercises can be repeated for mastery. An answer key for the odd exercises is included in the Appendix. Ensure that students have met the lesson objective before moving on.

The *Quizzes & Tests* evaluate student progress. Often open-response questions are utilized that require students to state, in writing, their understanding of the concept under examination. This is an invaluable skill that ensures that students understand the mathematical concepts they are learning rather than becoming mere computers. Throughout this course, there are 21 quizzes, one at the conclusion of each part, and five tests, one at the conclusion of each unit. Quizzes focus on the material in their respective parts, but exams are cumulative. There is also a Final Exam covering Units I-V that should be attempted after intensive review. Mastery of the material presented in this course should translate to a score of 85 or greater on the Final Exam. Reserve Form B assessments for retakes or for Final Exam review in the closing weeks of the course.

The *Teacher Manual* serves as the primary instructional resource for the course and includes an answer key for all lesson exercises. Before presenting a lesson to students, thoroughly review the material and the teacher notes for helpful techniques and emphases. A teacher or homeschool instructor with an elementary understanding of algebra will be able to convey the lesson concepts and achieve the lesson objectives by understanding and following the material included in the *Teacher Manual*. The *Solutions Key* contains complete, detailed solutions for every exercise to ensure comprehension.

A recitation schedule is located in the Appendix of the *Student Text* and *Teacher Manual*. Students should master the responses to the recitation questions for each lesson, reviewing recitation questions as necessary until they are mastered.

IS YOUR STUDENT READY FOR ALGEBRA I?

If there is any doubt regarding your readiness for this course, complete the Readiness Exam and grade it. If your student scores 85 or greater, you can be confident that he or she will succeed in this course. If your student scores between 80 and 84, you will want to review the topics or skills that are weak before beginning this course. A score below 80 likely indicates that your student does not have the foundation necessary to pick up the study of algebra where *Memoria Algebra I* begins. We would recommend completing *Memoria Pre-Algebra* prior to undertaking this course.

UNIT I

First-Degree Relations with Two Variables

We begin *Memoria Algebra I* by picking up where you left off in *Memoria Pre-Algebra* and building upon the skills you learned for solving first-degree relations with one variable by expanding your usage of those skills in relations with two variables. The concept of "relation" becomes fully formed in this unit as you will consider how the relationship (equality or inequality) between two variable quantities affects their respective values. You will explore these relationships both algebraically and graphically as you are introduced to René Descartes' seventeenth-century innovation known today as **analytical geometry**. Unit I concludes with real-world applications of these relations, further buoying our foundational assertion that the universe is a mathematical world.

PART A: THE SOLUTION SET FOR ONE OPEN SENTENCE

Solving equations results in solution sets, and as you further pursue the taming of the unknown in your study of algebra, you need to know how to state the solution set for first-degree relations with two variables. You can do so both algebraically – using set notation, with which you are already familiar – and graphically – using the Cartesian coordinate system, or the two-dimensional xy -plane, as opposed to the one-dimensional number line.

As you will shortly see, the solution set for a single first-degree relation with two variables always graphs as a straight line (linear). Thus, you will define slope and intercepts as the two most important and helpful characteristics of such graphical representations. It is important for you to seek to correlate algebraic expression and graphical representation in your understanding as different depictions of the same mathematical reality. The Cartesian coordinate system is helpful to the extent that you recognize its purpose: visual, geometric depiction of an algebraic reality. In short, you can express the solution set for a linear relation in either set notation or as the graph of a line in the Cartesian plane. The (x, y) coordinates of every point that makes up the line (and there are an infinite number of points!) represent a solution to the relation. For such infinite solution sets, a graph is a powerful tool of communication. A picture truly is worth a thousand words!

The beauty of both of these systems is that they enable clear, concise, and powerful expression of otherwise rather abstract concepts.

LESSON 1

The Cartesian coordinate system is the means of graphically depicting the solution set for relations in two variables. Thus, obtaining a strong foundation in these terms and concepts and obtaining the ability to graph ordered pairs in the plane with ease are imperative.

Work with your students on mastering the ability to label the parts of the Cartesian plane, including the quadrants, and to label the coordinates in an ordered pair. In the same way that they pursued and achieved number sense in arithmetic, so they must pursue and achieve a sense with reference to the Cartesian coordinate system.

A helpful way to relate quadrant numbers to ordered pairs is to show that all ordered pairs in quadrant I have positive x - and y -coordinates, i.e. $(+, +)$. In quadrant II, all ordered pairs are $(-, +)$. All the ordered pairs in quadrant III are $(-, -)$, and those in quadrant IV are $(+, -)$. Mastering these relationships gives students a quick method for reasonableness check regarding the accuracy of their graphs and their reading of ordered pair coordinates from graphs.

Continue to teach students that in order to graph an ordered pair, they must begin at the origin, move along the x -axis the same number of units as the x -coordinate in the proper direction (left for negative and right for positive), and then move from that point up (for positive) or down (for negative) the same number of units as the y -coordinate. This process is best taught via visual demonstration. Remember, consistency in methodology leads to mastery!

Lesson 1: Graphing Terminology

OBJECTIVE: Define, gain familiarity with, and master the terms used to describe the Cartesian coordinate system.

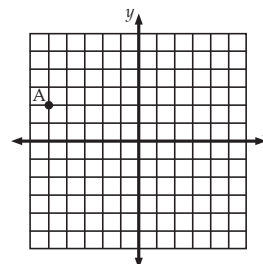
IMPORTANT TERMS

1. **Rectangular Coordinate System:** Also called the Cartesian coordinate system (named for its developer, René Descartes, a seventeenth-century French scientist and philosopher). It is a framework within which solutions for first-degree relations using two variables can be shown.
2. **Coordinate Axes:** The two number lines placed at right angles and intersecting at zero, which are used to name all the points in a plane.
3. **Horizontal Axis:** Often called the x -axis, the horizontal number line in a rectangular coordinate system, always associated with the first variable in an ordered pair.
4. **Vertical Axis:** Often called the y -axis, the vertical number line in a rectangular coordinate system, always associated with the second variable in an ordered pair.
5. **Origin:** The point $(0, 0)$ where the coordinate axes intersect. The origin is the reference point for every other point in the plane.
6. **Ordered Pair:** Often called the coordinates of a point, two numbers that locate and identify a point in the plane. The x -coordinate is always first, followed by the y -coordinate, and they are grouped using parentheses.
7. **Graph of an Ordered Pair:** The point in the plane that is located by, and identified with, a particular ordered pair.
8. **Abscissa:** From a Latin word meaning "to cut," the first number in an ordered pair, the number that tells us where to "cut" the horizontal axis as we begin locating a point. This is another name for the x -coordinate.
9. **Ordinate:** From a Latin word meaning "ordered," the second number in an ordered pair, the number that we use, in an "ordered" way, after we use the abscissa. This is another name for the y -coordinate.
10. **Quadrant:** Any one of the four sections of the rectangular coordinate system created by the coordinate axes. They are numbered for convenience beginning at the upper right (quadrant I) and moving in a counterclockwise direction (quadrants II, III, IV).

EXAMPLE 1

Complete the statements about the point A indicated in the following rectangular coordinate system.

- a. The ordered pair is _____.
- b. The abscissa is _____.
- c. The ordinate is _____.
- d. The point is in quadrant _____.
- e. The point is _____ units (left or right) from the origin.
- f. The point is _____ units (up or down) from the origin.



EXAMPLE 1 (continued)**Solution**

- a. The point A is 5 units to the left on the horizontal axis, and left is negative. It is also 2 units up on the vertical axis, and up is positive.
So the ordered pair is $(-5, 2)$, since the horizontal axis is always considered first.
- b. The abscissa is the first number in an ordered pair. In this case, the abscissa is -5 .
- c. The ordinate is the second number in an ordered pair. In this case, the ordinate is 2 .
- d. We know the upper right quadrant is quadrant I. Moving counterclockwise, the point is in quadrant II.
- e. From the explanation above, we know the point is 5 units to the left of the origin on the horizontal axis.
- f. From the explanation above, we know the point is 2 units up from the origin on the vertical axis.

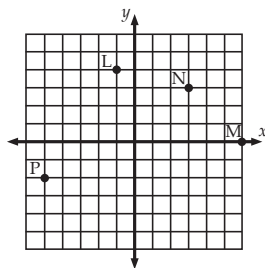
EXAMPLE 2

On the rectangular coordinate system given, plot the point indicated by each ordered pair, and label it appropriately.

- a. $L = (-1, 4)$ b. $M = (6, 0)$ c. $N = (3, 3)$ d. $P = (-5, -2)$

Solution

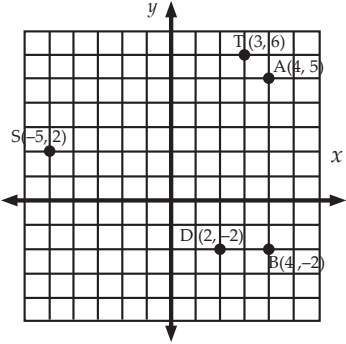
- a. The point L is 1 unit to the left of the origin and 4 units up from the origin.
- b. The point M is 6 units to the right of the origin and 0 units up or down from the origin. It is on the horizontal axis.
- c. The point N is 3 units to the right of the origin and 3 units up from the origin.
- d. The point P is 5 units to the left of the origin and 2 units down from the origin.



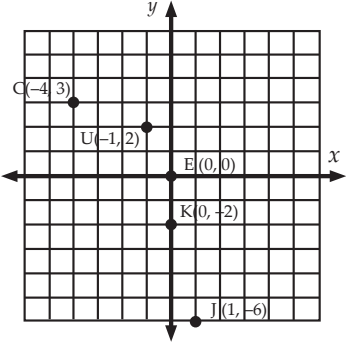
EXERCISE KEY

- 1. (3, 5)
- 2. 0
- 3. IV or 4
- 4. (6, -3)
- 5. 6
- 6. 0
- 7. 3
- 8. 3
- 9. -3
- 10. Q
- 11. (-4, 0)
- 12. 3
- 13. 5
- 14. I or 1
- 15. -4

16-20.

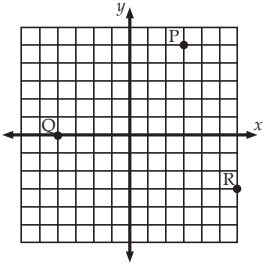


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EXERCISES

Complete the statements about the points indicated on the following rectangular coordinate system.



1. The ordered pair for point P is _____.

2. The ordinate for point Q is _____.

3. Point R is in quadrant _____.

4. The ordered pair for point R is _____.

5. The abscissa for point R is _____.

6. Point Q is _____ units up from the origin.

7. The abscissa for point P is _____.

8. Point P is _____ units to the right of the origin.
9. The ordinate for point R is _____.

10. Point _____ is on the horizontal axis.

11. The ordered pair for point Q is _____.

12. Point R is _____ units down from the origin.

13. The ordinate for point P is _____.

14. Point P is in quadrant _____.

15. The abscissa for point Q is _____.

On the same rectangular coordinate system, plot the point indicated by each ordered pair, and label it appropriately.

16. S is (-5, 2)

17. A is (4, 5)

18. D is (2, -2)

19. T is (3, 6)

20. B is (4, -2)

On the same rectangular coordinate system, plot the point indicated by each ordered pair, and label it appropriately.

21. U is (-1, 2)

22. E is (0, 0)

23. J is (1, -6)

24. C is (-4, 3)

25. K is (0, -2)

LESSON 12

The difference between this lesson and the previous lesson is the addition of a step in the process: Students must determine the slope of the graph of the equation using the slope formula and the two given points.

As in the previous two lessons, graphing the solution set here is rather straightforward. Students simply need to plot the two given points and draw the line through them.

When using the slope formula, ensure that students are careful to substitute the correct coordinates in the correct places in the formula. One helpful way to do this is to label the points as #1 and #2 and then label the individual coordinates accordingly. So, the first point would be (x_1, y_1) and the second would be (x_2, y_2) . The order or numbering of the given points does not matter; what matters is ensuring that the proper coordinates find their proper place in the slope formula. You may also need to re-draw and re-teach the slope triangle in the Cartesian plane so that the formula makes conceptual sense to your students.

Once students have determined the slope of the line, they can utilize the same procedures as in the previous lesson using one of the given points and **slope-intercept form** to determine the y -intercept and write an equation for the relation. And, as described in the previous lesson, **point-slope form** is an option at this point in the process as well.

Lesson 12: Given Two Solutions

OBJECTIVE: Write a relation that has a solution set line determined by two given solutions (points on the line).

IMPORTANT TERMS

1. **y -Intercept:** In the Cartesian coordinate system, the point where a solution set line intercepts the y -axis.
2. **Slope:** A term used to describe the slant of a line by comparing the vertical change of the line (the rise) to the horizontal change of the line (the run).
3. **Slope-Intercept Form:** The form of a first-degree equation that allows easy identification of the slope and y -intercept for the solution set line. The general form is $y = mx + b$, where m is the slope and b is the y -intercept.
4. **Standard Linear Form:** The form of a first-degree equation that has integer coefficients. The general form is $Ax + By = C$ where A , B , and C are integers.

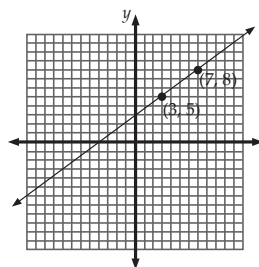
EXAMPLE 1

Write a relation in slope-intercept form that has a solution set line determined by the following pair of solutions (points on the line). Then convert the relation into standard linear form.

one solution is $(3, 5)$
another solution is $(7, 8)$

Solution

Plot the given solutions and draw the line containing them to graph the solution set.



Compute the slope between the two points by computing the vertical change (rise) and the horizontal change (run).

Moving from point $(3, 5)$ to point $(7, 8)$, we have a vertical change of 3 (from 5 to 8) and a horizontal change of 4 (from 3 to 7). This gives us a slope of $\frac{3}{4}$. Use the definition of slope as the ratio of rise to run as follows:

$$\frac{\text{rise}}{\text{run}} = \frac{8-5}{7-3} = \frac{3}{4}$$

EXAMPLE 1 (continued)

Note: The slope formula is designed to give you the slope of a line if you know two points on the line.

It is written as: $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are the coordinates of the two points on the line.

Write the desired equation in slope-intercept form as follows:

$$y = \frac{3}{4}x + b$$

You don't know what b is, but you do know that either of the given solutions will make the sentence true. So, choose one of the solutions, say $(3, 5)$, and substitute 3 for x and 5 for y . Then solve for b as follows:

$$\begin{aligned} (5) &= \frac{3}{4}(3) + b \\ 5 &= \frac{9}{4} + b \\ \text{Add } \frac{-9}{4} & \quad \frac{20}{4} + \left(\frac{-9}{4}\right) = \frac{9}{4} + b + \left(\frac{-9}{4}\right) \\ & \quad \frac{11}{4} = b + 0 \\ & \quad \frac{11}{4} = b \end{aligned}$$

So, the y -intercept (b) must be $\frac{11}{4}$.

Write the completed equation in slope-intercept form as follows: $y = \frac{3}{4}x + \frac{11}{4}$.

Convert this equation into standard linear form by multiplying both sides of the equation by 4.

$$\begin{aligned} \text{Mult. 4} \quad 4(y) &= 4\left(\frac{3}{4}x + \frac{11}{4}\right) \\ 4y &= \frac{12}{4}x + \frac{44}{4} \\ 4y &= 3x + 11 \\ -3x + 4y &= 11 \end{aligned}$$

Note: You may encounter a formula that is designed to give you an equation for a solution set line if you know two solutions (points on the line). It is called the **two-point form** and it is written as:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

where (x_1, y_1) and (x_2, y_2) are the two given points. This formula is valid, but in this course we will focus on proficiency with slope-intercept, point-slope, and standard linear form.

EXERCISE KEY

1. $y = 1 \cdot x + 1$

$$-1x + 1y = 1$$

2. $y = -2x + 0$

$$2x + 1y = 0$$

3. $y = \frac{5}{2}x + 6$

$$-5x + 2y = 12$$

4. $y = \frac{1}{5}x + 2$

$$-1x + 5y = 10$$

5. $y = \frac{-1}{2}x + (-1)$

$$-1x - 2y = 2$$

6. $y = 3x + (-2)$

$$-3x + 1y = -2$$

7. $y = \frac{3}{-5}x + 0$

$$-3x - 5y = 0$$

8. $y = 0 \cdot x + 5$

$$0x + 1y = 5$$

9. "No slope." So our equation is $x = 1$ where the line will be vertical. No restrictions are placed on y .

10. $y = 1 \cdot x + 3$

$$-x + y = 3$$

EXERCISES

For each of the following, write a relation in slope-intercept form that has a solution set line determined by the two given points. Then convert the relation into standard linear form.

1. two solutions are: (2, 3) and (5, 6)

6. two points on the line are: (2, 4) and (3, 7)

2. two solutions are: (0, 0) and (-1, 2)

7. two solutions are: (5, -3) and (0, 0)

3. two points on the line are: (-4, -4) and (-2, 1)

8. two points on the line are: (0, 5) and (3, 5)

4. two points on the line are: (-5, 1) and (0, 2)

9. two solutions are: (1, 3) and (1, 7)

5. two points on the line are: (0, -1) and (4, -3)

10. two points on the line are: (1, 4) and (3, 6)