

The CHARLOTTE MASON
JUNIOR MATH SERIES

Pre-algebra



Art Middlekauff

The Charlotte Mason Junior Math Series

Pre-Algebra

by Art Middlekauff

The Charlotte Mason Junior Math Series: Pre-Algebra
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ISBN 978-1-61634-718-5 printed
ISBN 978-1-61634-719-2 electronic download

Published by
Simply Charlotte Mason, LLC
930 New Hope Road #11-892
Lawrenceville, Georgia 30045
simplycharlottesmason.com

Printed in the U.S.A.

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Introduction¹

Mathematics is an intrinsic element of the Charlotte Mason method, and as with every other subject, the “three educational instruments” we employ are “the atmosphere of environment, the discipline of habit, and the presentation of living ideas.”² However, unlike other subjects, the habits of math are formed in the mind where they are rarely observed, and the living ideas are introduced by a living teacher rather than by a living book. This book is designed to help you be such a teacher to your student.

When Charlotte Mason speaks of ideas, she does not mean symbols, techniques, or procedures. She means something that is active:

“An idea is more than an image or a picture; it is, so to speak, a spiritual germ endowed with vital force—with power, that is, to grow, and to produce after its kind. It is the very nature of an idea to grow: as the vegetable germ secretes that [which] it lives by, so, fairly implant an *idea* in the child’s mind, and it will secrete its own food, grow, and bear fruit in the form of a succession of kindred ideas.”³

As hard as it may be for some to believe, mathematics is filled with such ideas. Ideas that do not merely direct the computer-like manipulation of symbols according to rigorously defined rules and procedures. Rather, it is ideas that excite the imagination and prompt a “succession of kindred ideas.”

An extremely important aspect of teaching math the Charlotte Mason way, then, is to identify the root ideas that organically prompt the rest. She referred to these as “guiding” or “captain” ideas. According to Mason, math instruction without these captain, guiding, living ideas falls horribly short of the wondrous adventure that mathematics should be:

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1. This introduction is based on the 2020 article “Math for Older Students” by Richele Baburina and Art Middlekauff.
 2. C. M. Mason (1925), *A Philosophy of Education*, p. xxix.
 3. C. M. Mason (1935), *Home Education*, p. 173.

“Mathematics depend upon the teacher rather than upon the textbook and few subjects are worse taught; chiefly because teachers have seldom time to give the inspiring ideas, what Coleridge calls, the ‘Captain’ ideas, which should quicken imagination.”⁴

One purpose of this book is to *give* you the time to *give* the inspiring ideas to your children. The book is organized around captain ideas, which give meaning and relevance to the details and applications that follow. Every effort has been made to weave in interesting information about the history of these ideas to further excite the imagination.

Now, as important as ideas are, it is essential to realize that ideas must be *consumed*. They cannot be *pushed in*. Just as with every other subject in a Charlotte Mason education, the student must *work* to grasp the living idea. There must be *self-activity*. Very few children will grasp a living idea by watching a brilliant lecture or demonstration, either live or on video. Instead, children must work to *meet* an idea. They must make their own *discovery*, differing only in degree and not in kind to the original discoveries by mathematicians of the past.

Discovery in Charlotte Mason math doesn’t mean the child is left to wander aimlessly. Rather, the teacher makes sure to give her the tools and vocabulary necessary while at the same time being sure the student is the one doing the work. The way this works out in practice is typically a conversation or dialog between the teacher and the student. Each lesson in this book is essentially an outline or framework for a conversation of discovery. The words don’t need to be used verbatim, and in fact it is good to deviate a bit from the script when your student has questions or other needs.

While definitions and formulas are *memorized*, ideas are *grasped*. When the “light bulb” goes on, the student has *understanding*. This text offers no neat rules or definitions to commit to memory. Rather, this book is an aid to help you bring your student to the point of understanding at which no rote repetition is required.

Nevertheless, although ideas form the first part of each lesson, they are not the last part. Habit is the second instrument of a Charlotte Mason

4. Mason (1925), p. 233.

education, and she explains that “a habit is set up by following out an initial idea with a long sequence of corresponding acts.”⁵ The living ideas must be worked on by the child until they “leave their mark upon the quite palpable substance of the brain.”⁶ As with elementary math in the Charlotte Mason method, this is done by following the formula “new, review, mental math too.” New ideas are immediately worked on via exercises and problems. And then review and mental math complete the path to fluency. By mental math, we mean work that is taken orally.

To support the formation of habits, this book includes written exercises, oral exercises, and review exercises. Many of these exercises are progressive in nature, and most students would benefit from doing all of them. Even when a student fully understands the concepts, it is still necessary to form the mental habits by doing these exercises.

The third instrument of education is the atmosphere of environment. By this, Mason “do[es] not mean that a child should be isolated in what may be called a ‘child-environment’ especially adapted and prepared, but that we should take into account the educational value of his natural home atmosphere, both as regards persons and things.”⁷ In elementary math, that means using real-world objects in the place of contrived manipulatives. In junior math, that also means connecting math ideas to elements of the real world. Therefore this book introduces math concepts by relating to familiar objects such as thermometers and maps. No artificial or abstract manipulatives are needed since math can be found in natural objects all around the home.

One of Charlotte Mason’s many gifts was her ability to discern a small number of principles that have broad and overarching application. Atmosphere, discipline, and life, which are the three instruments of education, are as relevant to the teaching of math as they are to the teaching of history, literature, and science. Your awareness of how these principles play out in the math lesson will enable you to benefit more from the structure and content of this book.

5. C. M. Mason (1904), *Parents and Children*, p. 125.

6. *Ibid.*, p. 123.

7. Mason (1925), p. xxix.

Prerequisites

The Charlotte Mason Junior Math Series is intended for students who have mastered elementary arithmetic. That may be summarized as the four basic operations (addition, subtraction, multiplication, and division) applied to whole numbers, fractions, and decimals. A student who has completed the Charlotte Mason Elementary Arithmetic Series will have an excellent foundation for this book. This first book in the Junior Math Series functions as a bridge between arithmetic and algebra. It introduces concepts such as negative numbers, which were considered to be algebra in Charlotte Mason's time, but which are now often considered to be advanced arithmetic. This book also introduces formulas, which lay the groundwork for variables and solving for the unknown in algebra.

This book does not include an explicit review of elementary arithmetic. However, there is an implicit review in that lessons from the beginning of this book include whole numbers, fractions, and decimals. If your student experiences difficulty with any of the exercises involving fractions or decimals, you are encouraged to go back to these topics in the Elementary Arithmetic Series for a brief review before continuing with the pre-algebra text.

How To Use This Book

This book is intended primarily for you as a teacher and not directly for your student. Normally your student will refer to the text only when working independently on the written exercise sections. Generally, even the oral exercises will be done without the student looking at the book, although some will refer to an illustration such as a chart which will need to be viewed to complete the exercise. Each lesson begins with a guide, which is a suggested script to help your student discover the main ideas of the lesson. You should read this script in its entirety before beginning the lesson with your student. You may also wish to look ahead to see what supplies will be needed for the next few lessons.

The lesson guides make frequent reference to a slate. This can be a personal chalkboard, dry-erase board, or the like, together with its appropriate writing instrument. It is the most important accessory for your math lessons. If teaching in a home setting, you can sit side-by-side with your student and together work on the slate or dry-erase board as you illustrate concepts as they emerge during the lesson. The lesson guides suggest what to write on the slate and when.

This book does not prescribe a suggested duration for lessons. This is intentional, because the pace should be based on the rate of your student's learning. The pace should not be so fast that it frustrates your student, nor should it be so slow that it bores your student. Enough time should be spent on the lesson for your student to get comfortable with the main ideas and successfully complete the exercises, but no more time than that. Vyvyan Richards summarized this approach well in 1955:

“Keep the work *relevant*, suited to the child's power of understanding. Give him a programme ... easy enough for the confidence to return which Miss Mason wanted, and give him a sense of mastery—and, no doubt, with it the assurance that this teacher can teach after all! Or let the [work] chosen

be easy enough for some to get it all right, and none to feel defeated and silly; for that is to offend against their integrity.”⁸

In some cases, your student may need more practice than provided by the oral and written exercises in this book. If that is the case, you are encouraged to make up your own exercises using the ones provided in the text as models.

The end of the book provides three tests. Charlotte Mason viewed examinations as essential to the learning process. They were not principally for the educator but for the student. In *A Philosophy of Education* she wrote:

“Knowledge is acquired only by what we may call “the *act of knowing*,” which is both encouraged and tested by narration, and which further requires the later test and record afforded by examinations.”⁹

In Charlotte Mason math there is no “narration,” but there are “acts of knowing,” which occur whenever a student does oral and written exercises. Mason’s point is that a second round of “knowing” occurs when a student takes examinations at the end of a term. Although there is no special studying or “revision” before a test, the fact of an end-of-term examination helps the student to naturally retain what she has learned in her long-term memory.

The tests included in this book, however, should not be used to force a learning pace on the student. If a term ends before the student has completed the lessons, then the test should be adjusted to cover only the material the student has mastered.

8. *The Parents’ Review*, volume 66, p. 128.

9. Mason (1925), p. 291.

Concepts in Junior Math Pre-Algebra

Signed Numbers

- Meaning
- Comparing Signed Numbers
- Adding Signed Numbers
- Subtracting Signed Numbers
- Multiplying Signed Numbers
- Dividing Signed Numbers

Solving Expressions

- Order of Operations
- Parentheses and Terms
- The Distributive Law

Powers and Roots

- Meaning
- Calculating
- The First and Zeroth Power
- Powers of Ten
- Exponents and Radicals

Formulas

- Variables and Rules
- Formulas with Addition and Subtraction

- Formulas with Multiplication and Division
- Formulas with Powers and Roots
- Formulas for Finding Area
- The Speed Formula

Data Analysis

- Tables of Data
- Line Charts
- Bar Charts
- Other Kinds of Charts
- Mode, Median, and Mean

Supplies Needed

The two most important supplies required to use with this book are the **math notebook** and the **slate**. The math notebook is where the student will be doing her written work. In order to keep written work as neat as possible, $\frac{1}{2}$ " or $\frac{1}{4}$ " gridded graph paper is desirable, and it is essential for the section on charting. In the back of the notebook, the student should have a section to write rules, steps, and formulas that may serve as an ongoing reference. If you have been using the Charlotte Mason Elementary Arithmetic Series, it is ideal to continue using the same notebook. If the student has written down rules for area and unit conversions in the back of her notebook, she will be able to use that information in this course.

The slate can be a personal chalkboard, dry-erase board, or the like, together with its appropriate writing instrument. This will be used during the lesson discussion and much of the oral work.

The following are the other objects required for various lessons in this book:

- an analog thermometer which includes both positive and negative temperatures or, alternatively, a picture of such a thermometer
- a globe that shows latitude and longitude or, alternatively, a flat map that shows latitudes and longitudes for both hemispheres
- coin purse containing 3 dollar bills, 3 nickels, 6 dimes, and 3 quarters
- paper cutout of a parallelogram
- blank piece of paper
- scissors
- a calendar, planner, or journal
- protractor
- compass (drawing tool)
- colored pencils

Signed Numbers¹⁰

A New Kind of Number

(instructions to the parent)

Objects used: analog thermometer which includes both positive and negative temperatures or a picture of such a thermometer; globe that shows latitude and longitude or a flat map that shows latitudes and longitudes for both hemispheres

Read through this entire section, including the oral work, so that you are familiar with it and are ready for any questions that might come up. This lesson defines a *signed number* as “a number with two components: a *sign* and a *magnitude*.” These terms will be used throughout these lessons, so please use them consistently. If you or your student is unfamiliar with the word *magnitude*, you can explain that it means “size” or “quantity.” The magnitude of a number is a measure of how far it is from zero. For example, -5 and $+5$ are the same distance from zero but in opposite directions. Both have a magnitude of 5. While it is important that you use the terms *sign* and *magnitude*, feel free to put the rest of this lesson into your own words and to spread this lesson out in as many daily lessons as necessary.

1. Ask: Could you show me the number 10 on this thermometer? (The student may point to $+10$ or -10 .)
2. Point to the other 10 and ask, Is not this also 10? Why are there two 10s on this thermometer? Invite the student to explain to you in her own words based on her own understanding of temperature. She may use words like “above zero” and “below zero,” or even “negative

10. I give credit to Clifford Upton, who used this phrase as the subtitle for Chapter IV of his *Practical Algebra Introductory Course* (1936) in his chapter on “Signed Numbers.”

10.” It is important to allow her to articulate her own understanding of this familiar concept, as this will help her clarify her own thoughts.

3. Ask: Why do our temperature scales allow values to be “below zero”? The student may or may not be able to answer this. If you are looking at a Celsius thermometer, you may explain that the Celsius scale was designed such that 0° is the freezing point of water. For Fahrenheit, you may explain that 0° was selected as the freezing point of salt water. The reality with both scales is that temperatures can go much lower than those freezing points.
4. Explain: Whenever you are working with a scale that allows values to be “below zero,” you need a new kind of number. Up until now in our study of arithmetic, numbers have represented only a *magnitude*, that is, a size or quantity. However, for devices like a thermometer, we need a new kind of number that has not only a magnitude but also a direction. We call that direction the *sign*, and we call this new kind of number a *signed number*. For a thermometer, a temperature above the freezing point of water (for Celsius) or salt water (Fahrenheit) is said to be *positive*. A temperature below the freezing point is said to be *negative*.
5. Now get out the globe or map and ask: Could you show me the 10° latitude line? The student may point to 10° North (N) or 10° South (S).
6. Point to the other 10° latitude (N or S) and explain: Just as a thermometer has values above and below zero, the latitude and longitude scales have magnitudes above and below zero. A latitude above the Equator is said to be *positive*. A latitude below the Equator is said to be *negative*. Similarly, a longitude east of the Prime Meridian is positive, and a longitude west of the Prime Meridian is negative. A value in the latitude or longitude scale includes both a magnitude and a direction. That direction can either be specified as a cardinal direction (N or S) or as a sign (positive or negative).

7. Explain: When we write down a signed number, we use the same symbols that we use for addition and subtraction: the plus sign and the minus sign. However, even though the symbols are the same, the meaning is different. Until now, we have used the plus and minus signs only as *signs of operation*. With signed numbers, however, we use these symbols as *signs of quality*. The sign is part of the number itself.¹¹
8. Write the number -5 on the slate and explain that we say this as “negative five.” Write the number $+6$ on the slate and explain that we say this as “positive six.”
9. Ask: Do you think the number 0 can be a signed number? Why or why not? If the student is not sure, ask her if it makes sense to say the temperature outside is “zero above zero” or “zero below zero.” Since the sign indicates a direction relative to 0, the number 0 itself can’t have a sign.

Oral Work

1. Write in a column on the slate or on grid paper and ask your student to read:

-3 (negative 3)	$+10$ (positive 10)	$+3$ (positive 3)
-15 (negative 15)	-6 (negative 6)	$+1$ (positive 1)
-14 (negative 14)	$+16$ (positive 16)	$+12$ (positive 12)
-10 (negative 10)		

11. Upton, p. 109.

2. Ask your student to write in a column, either on the slate or in her math notebook, while you dictate:

negative four (-4)

positive eight ($+8$)

positive three ($+3$)

negative eleven (-11)

negative ten (-10)

negative two (-2)

positive twenty ($+20$)

negative fourteen (-14)

positive nineteen ($+19$)

positive eleven ($+11$)

3. Write in a column on the slate or on grid paper and ask your student to read:

$-\frac{3}{5}$ (negative three-fifths)

$-\frac{2}{3}$ (negative two-thirds)

$+\frac{1}{10}$ (positive one-tenth)

$+\frac{1}{2}$ (positive one-half)

$+3\frac{3}{4}$ (positive three and three-fourths)

$-\frac{3}{8}$ (negative three-eighths)

$-\frac{5}{4}$ (negative five-fourths)

-0.5 (negative five-tenths)

$+1.54$ (positive one and fifty-four one-hundredths)

$+12.82$ (positive twelve and eighty-two one-hundredths)

4. Ask your student to write in a column either on the slate or in her math notebook while you dictate:

positive two-sevenths ($+\frac{2}{7}$)

positive one-third ($+\frac{1}{3}$)

negative nine-tenths ($-\frac{9}{10}$)

negative two and one-half ($-2\frac{1}{2}$)

negative five-halves ($-\frac{5}{2}$)

positive four-fifths ($+\frac{4}{5}$)

positive eleven-twelfths ($+\frac{11}{12}$)

negative fourteen and one-tenth (-14.1)

positive twenty-six and twenty-four one-hundredths ($+26.24$)

positive six and two one-hundredths ($+6.02$)

5. Say each of the following latitudes and longitudes, and after each one, ask your student to restate it as a positive or negative number:

30° N ($+30^\circ$)

178° S (-178°)

19° N ($+19^\circ$)

114° S (-114°)

134° S (-134°)

82° E ($+82^\circ$)

51° W (-51°)

135° E ($+135^\circ$)

67° E ($+67^\circ$)

98° W (-98°)

6. Ask: How would you say the temperature in Celsius as a signed number when it is 5 degrees below the freezing point of water? (negative five degrees Celsius)
7. Ask: How would you say the temperature in Fahrenheit as a signed number when it is 32 degrees above the freezing point of salt water? (positive thirty-two degrees Fahrenheit)

8. Ask: If I had a bank account and the balance was $-\$3$, what do you think that might mean? (Answers may vary, but will sound something like: *It means that you have a debt; you owe the bank money.*)

Tip: Don't worry if the student is not able to answer this question. If she does not have an answer, then this is a great opportunity to explain that positive and negative numbers are commonly used to express whether you have or owe money.

9. Ask: If an elevator had a button to go to floor -2 , where do you think that might take you? (One level below the first basement, or two floors below the main floor.)
10. Ask: It is said that Bombay Beach in California has an elevation of -68 m (negative sixty-eight meters). What do you think that means? (It means 68 meters below sea level.) How would we express the elevation of Denver, which is 1,673 meters above sea level? (The elevation of Denver is positive 1,673 meters.)

Tip: This is a great opportunity to again explain that the sign of the number indicates the direction (above or below sea level) and the magnitude of the number indicates how far in that direction.

Comparing Signed Numbers

(instructions to the parent)

Objects used: analog thermometer which includes both positive and negative temperatures or a picture of such a thermometer; globe that shows latitude and longitude or a flat map that shows latitudes and longitudes for both hemispheres

Read through this section so that you are familiar with it and feel free to put it into your own words.

1. Say: When we are working with temperature, we say that a high temperature is *greater than* a low temperature, and a low temperature is *less than* a high temperature. *High* refers to the degree of heat, which also corresponds to how high the temperature is on the thermometer.
2. Ask: With this in mind, how would you compare 10° to 20° ? (The student should say that 10° is less than 20° , and/or that 20° is greater than 10° . If the student does not say this, encourage her to compare the temperatures using the *greater than* / *less than* terminology.)
3. Ask: By looking at your thermometer, would you say that 5° is greater than or less than -10° ? (The student should say that 5° is greater than -10° .) You may want to point out that it can be a bit surprising when a number with a greater magnitude (-10) is actually less than a number with a smaller magnitude (5).
4. Ask: Are negative numbers greater than or less than zero? Are positive numbers greater than or less than zero? (The student should answer that all negative numbers are less than zero and all positive numbers are greater than zero.)
5. Say: When comparing signed numbers, you can always think of temperatures, in which “greater than” means “warmer than.” You may point out that it is easier to just think of a thermometer than to memorize certain rules.

Oral Work

1. Ask: Fairbanks, Alaska, has an average monthly minimum temperature of -16.9°F , while Grand Forks, North Dakota, has an average monthly minimum temperature of -3.1°F . Which city has the greater temperature on average? (Grand Forks, North Dakota)

2. Ask: Which city is farther north, Albuquerque at latitude $+35^\circ$ or Albany at latitude $+42^\circ$? (Albany)
3. Ask: If an elevator has a level -3 and a level -4 , which is the higher basement level? (level -3)
4. Write the following pairs of numbers on the slate. For each pair, ask the student to say which is the greater value.

$-11, +28 (+28)$

$-10, -25 (-10)$

$+20, -1 (+20)$

$-13, -6 (-6)$

$-16, +26 (+26)$

5. Write the following pairs of numbers on the slate. For each pair, ask the student to say which is the lesser value.

$-32, +17 (-32)$

$+18, -8 (-8)$

$-2, -28 (-28)$

$-22, -16 (-22)$

$+34, +5 (+5)$

6. Write the following pairs of numbers on the slate. For each pair, ask the student to say which is the greater value.

$+\frac{1}{2}, +\frac{1}{4} (+\frac{1}{2})$

$-\frac{1}{2}, +\frac{1}{4} (+\frac{1}{4})$

$-\frac{1}{2}, -\frac{1}{4} (-\frac{1}{4})$

$+0.5, +0.25 (+0.5)$

$-0.5, +0.25 (+0.25)$

$-0.5, -0.25 (-0.25)$

Radicals

(instructions to the parent)

It is worthwhile to take the time to explain to your student the meaning and history of the radical sign. It's an interesting story!³¹ Also, it reinforces for your student the relationship between radicals and roots. I remember that when I was a child, the radical sign to me looked like a modified long division symbol. I got the impression that radicals were a fancy kind of division. It was hard for me to shake that notion! I wish I had been taught what the symbol actually represented.

1. Say: Up till now we have been talking about finding the root of a number. However, we haven't talked about a compact way to write it. Suppose we started with this as a way to express the *square root* of 4. Write on the slate, taking care to use a capital *R*:

Root of 4

We could abbreviate that by just using an *R* for *Root*. Write on the slate:

R 4

This is just what mathematicians of prior centuries did. However, they wanted to make the *R* look special so that people understood that it was an abbreviation for the word *root*. We believe that over time, that fancy *R* changed so much that it no longer looks like an *R*. Instead, amazingly enough, it became a shape that looks like a check mark. Write the radical sign on the slate, but without the horizontal bar over the 4, like this:

$\sqrt{4}$

31. The history of the radical sign presented here is adapted from *A History of Mathematical Notations* by Florian Cajori, pp. 366–379, originally published 1928–1929.

However, there is a limitation with this check-mark symbol. Sometimes you want the square root of an entire expression. Write on the slate:

$$\sqrt{4 + 4}$$

Is this supposed to be the square root of 4, or the square root of $4 + 4$? To avoid that confusion, mathematicians later added a bar, called a *vinculum*, to the top of the symbol. Write on the slate:

$$\sqrt{4 + 4}$$

Today this check-mark symbol with the bar on top is known as a *radical sign*. You may wonder why we call it a “radical sign” instead of a “root sign.” Actually, our word *radical* comes from the Latin word *radicalis*, which means “pertaining to the root.”³² So when you see the radical sign, you can think of it as the Latin abbreviation for “the root of.”

2. Say: But what if we want a different root besides the square root, such as the cube root? For any root other than the square root, we add a little number in the left corner of the radical sign. It looks a lot like an exponent, except it is on the left instead of the right. We call it the *index*. Write on the slate:

$$\sqrt[3]{8}$$

Say: That is how we write the cube root (the third root) of 8. The radical sign, with the index, helps us to see how roots are the opposite of powers. Write on the slate:

$$2^3 = 8$$

$$\sqrt[3]{8} = 2$$

3. Ask: How would you use the radical sign to write the fourth root of 16? Invite your student to write an answer on the slate. If she is

32. A. Lo Bello (2013), *Origins of Mathematical Words*, p. 269.

not sure, then write the answer yourself on the slate and continue to explain the notation:

$$\sqrt[4]{16}$$

Say: Remember that when we just want the square root (the second root), we don't write an index.

4. Say: There is one very important thing to know about the radical sign.

Ask: Do you remember how many square roots there are of a positive number? Your student may remember that every positive number has two square roots. For example, the square roots of 9 are 3 and -3. Say: Well, the radical sign refers only to the *positive square root*. Write on the slate:

$$\sqrt{9} = 3$$

Say: Even though we know that -3 is also a square root of 9, when we use the radical sign, we ignore that fact. You may want to ask your student if she thinks that is a good idea or not. On the one hand, it might bother her that this symbol seems to leave out a perfectly valid root. On the other hand, you can point out that it makes the math simpler to know that one symbol refers to only one number.

5. Say: Remember that odd-numbered roots of a negative number are negative. The radical sign refers to a negative number in these cases. For example: write on the slate:

$$\sqrt[3]{-27} = -3$$

But for even-numbered roots, the radical sign only refers to the positive root:

$$\sqrt[4]{16} = 2$$

The number under the radical sign is called the *radicand*, another word which we get from Latin, and has come to mean “a number whose root must be taken.”³³ When the radical sign, the radicand, and the index (if any) are all together, we call the whole thing a *radical*.

Oral Work

Write the following radicals on the slate and ask the student to solve them:

1. $\sqrt{4}$ (2)

2. $\sqrt{25}$ (5)

3. $\sqrt{36}$ (6)

4. $\sqrt[3]{8}$ (2)

5. $\sqrt[3]{-8}$ (-2)

6. $\sqrt[4]{16}$ (2)

7. $\sqrt{-9}$ (no real root)

Written Work

Write the answers as radicals. Do not solve the radicals.

1. If a square has an area of 196 cm^2 , what is the length of each side?
2. If a cube has a volume of $3,375 \text{ in.}^3$, what is the length of each side?
3. How would you represent a number that, when three instances of it are multiplied together, the product is -512 ?
4. Suppose we had a chain letter that reached 6,561 people in its fourth round. How would you represent the number that was given in the original instructions of the chain letter?

33. A. Lo Bello (2013), *Origins of Mathematical Words*, p. 269.

5. After 3 years, my garden has 2,197 weeds. How would you represent the number that shows how many weeds it had in the first year, assuming that each weed produced that same number of new weeds each year?

Answers are on page 29 of the Solutions Guide.

Review

(to be taken orally)

1. What is the square of -4 (negative 4)? (16)
2. What is the cube of -3 (negative 3)? (-27)
3. How many square roots does 16 have — one, two, or none? (two, 4 and -4)
4. How many square roots does -16 (negative 16) have — one, two, or none? (none)
5. How many cube roots does 27 have — one, two, or none? (one, 3)
6. How many cube roots does -27 (negative 27) have — one, two, or none? (one, -3)
7. Write on the slate: $\sqrt{16}$. How many answers does this have — one, two, or none? (one, 4)
8. Write on the slate: 3^4 . What is the base, what is the exponent, and what is the power? (The base is 3, the exponent is 4, and it is the fourth power.)

Review

(to be worked in the math notebook)

Evaluate the following expressions, remembering that an exponent applies only to the number that directly precedes it. Also remember to always multiply before adding or subtracting.

1. $1^3 + 2^2 + 3^1 + 4^0$

2. $(-1)^3 + (-2)^2 + (-3)^1 + (-2)^0$
3. $(-2)^3 + (-2)^2 + (-2)^1 + (-2)^0$
4. $-3^2 \times 5 - 2^3 \times 5$
5. $3 \times 10^2 + 6 \times 10^1 + 5 \times 10^0$
6. $\left(\frac{1}{3}\right)^2 + \frac{1^2}{9} + \frac{1}{3^2} + \left(\frac{1}{9}\right)^0$
7. $-2 \times \left(\frac{1}{2}\right)^2 + 2 \times \left(-\frac{1}{2^2}\right)$
8. $\frac{(-2)^4}{16} - \frac{-2^4}{8}$
9. $0.25^0 + 0.25^1 - 0.5^2$
10. $0.3^1 + 0.3^2 + 0.3^3$

Answers are on page 29 of the Solutions Guide.

Exponents and Roots in Expressions

(instructions to the parent)

This lesson introduces the final principles needed to evaluate even the most complicated expressions. Some of these expressions can look quite intimidating. However, you can reassure the student that by following a few simple rules and approaching the expression one step at a time, even the most intricate expression can be evaluated. Many students enjoy the feeling of competency that comes from being able to handle expressions that previously may have seemed intractable.

1. Say: We have now learned several symbols that can occur in expressions, such as radical signs and exponents. What happens when we mix them together? For example, how do we solve a very complicated expression like this? Write on the slate:

$$\sqrt[3]{31 - 2^2} + 3 \times \frac{4^2}{36 - 32} + 2 \times (4 - \sqrt{9})^2$$

Data Analysis

Introduction to Data

(instructions to the parent)

Objects used: a calendar, planner, or journal, the student's math notebook. For these lessons, $\frac{1}{2}$ " or $\frac{1}{4}$ " gridded graph paper is desirable.

If your student is involved in nature study or observation, that can be a great way to introduce the concept of data. You may want to personalize this opening section to make it more relevant to your child's actual experience with nature. During this lesson, it is recommended that you write the temperatures in an actual calendar, planner, or journal. You can use the temperature values supplied here or values you measure yourself. By writing the values in a calendar or planner first, it illustrates some of the differences between a calendar and a table. Make sure your student records this lesson's table in her notebook, because the table will be used in later lessons.

Using words like *observation*, *datum*, and *data* consistently will help your student learn the meaning of these words through a natural context rather than by memorization.

1. Say: Suppose we kept a record of outdoor temperature each day. To do so, we would need a thermometer to measure the outdoor temperature. Each day at a regular time, we would take an *observation* from the thermometer. We would read a number which would be *given* to us by the thermometer. The Latin word for "something given"

is *datum*. We might record the first day's datum on the calendar as follows. Write in your calendar or planner:

June

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1 82°	2	3	4	5	6	7

2. Say: As the week progresses, we make more observations. We record each datum on its appropriate day. Write in your calendar or planner:

June

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1 82°	2 72°	3 73°	4 73°	5 77°	6 79°	7 81°
8 81°	9 82°	10 95°	11 91°	12 84°	13 73°	14 77°

Say: Now we have 14 observations. The plural of datum is *data*. It may sound odd to some people, but *data* takes a plural verb. So we say, "The data *are* interesting." We might also say, "We have recorded data about the weather."

3. Say: We have recorded our observations in a calendar. Another way to record data is in a *table*. A table arranges observations in rows and columns. Our word *row* comes from the German word for *line*, and our word *column* comes from the Latin word for *pillar*. Columns do look a bit like pillars! In our case we will have 14 rows (1 for each day) and 2 columns (date and temperature). Help your student write a table in her math notebook based on the calendar. Note that space should be left on the page for a third column to be added to this table later in the lesson. When completed, the table should look like the following:

Date	Temperature
June 1	82°
June 2	72°
June 3	73°
June 4	73°
June 5	77°
June 6	79°
June 7	81°
June 8	81°
June 9	82°
June 10	95°
June 11	91°
June 12	84°
June 13	73°
June 14	77°

Ask: Looking at just the table, what is the highest temperature recorded in the two-week period? (The answer is 95°.)

Ask: Now looking at just the calendar, what was the lowest temperature recorded? (The answer is 72°.)

Ask: Was it easier to compare the temperatures using the calendar or using the table? (There is no right answer, but many people find the table easier to use when comparing values.)

Ask: The calendar and the table have the same data. But is there any situation in which the calendar format might be more useful? Is there any situation in which the table format might be more useful? Encourage your student to think about how both formats can be useful. For example, the calendar can be especially useful if you want to find the temperature on a particular day of the week. On the other hand, the table lets you see all the temperatures in a single glance.

4. Say: Now suppose we also used a rain gauge to measure rainfall each day. Now we would have two observations per day. Our calendar might look like the following. Add the rainfall data to your calendar or planner:

June

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1 82° 5 mm	2 72° 1 mm	3 73° 0 mm	4 73° 0 mm	5 77° 0 mm	6 79° 4 mm	7 81° 15 mm
8 81° 13 mm	9 82° 2 mm	10 95° 1 mm	11 91° 0 mm	12 84° 0 mm	13 73° 1 mm	14 77° 5 mm

5. Say: We can also add these rainfall data to our table. In a table, each row represents an item and each column represents an attribute of that item. In our table, the item for each row is a day. Now we have two columns of weather data because we have two observations for each day. Help your student add a third column to the table in her math notebook. When completed, the table should look like the following:

Date	Temperature	Rainfall
June 1	82°	5 mm
June 2	72°	1 mm
June 3	73°	0 mm
June 4	73°	0 mm
June 5	77°	0 mm
June 6	79°	4 mm
June 7	81°	15 mm
June 8	81°	13 mm
June 9	82°	2 mm
June 10	95°	1 mm
June 11	91°	0 mm
June 12	84°	0 mm
June 13	73°	1 mm
June 14	77°	5 mm

Say: Remember that the data in the calendar is exactly the same as the data in the table. We have just arranged the data in a different way.

Oral Work

Use the table in your notebook to answer the following questions:

1. What day had the most rainfall? (June 7)
2. How many days had no rainfall? (5 days)
3. What temperature occurred the most often? (73°)
4. How much rain was there on the hottest day? (1 mm)
5. How much rain was there on the coldest day? (1 mm)
6. If you were to make additional kinds of weather observations each day, what other columns might you add to a table? (Answers will vary. Some ideas include wind speed, humidity, and barometric pressure.)

The next several questions ask about other kinds of data.

7. If you were to measure the heights and weights of each person in your family and record the data in a table, what would each row represent? What would the columns be? (The general rule is in a table, each row represents an item and each column represents an observation about that item. Height and weight are observations about family members. So each row represents a family member, and the columns are height and weight. There may also be a column for identification that has the family member's name.)
8. Suppose you used a typing app to practice and test your typing skills. Each time you complete a practice session, the app displays your average words per minute and your percentage accuracy. If you were to record this data in a table, what would the rows and columns be? (Each row represents a practice session, and the columns are words per minute and percentage accuracy. There may also be a column for identification with the date and time of the practice session)

9. Imagine that you kept a record of all the nature trails near your home. For each trail, you record the length of the trail, average elevation, and how long it takes you to complete. How might you arrange this data in a table? (Each row represents a trail, and the columns are the length of the trail, the average elevation, and how long it takes to complete. There may also be a column with the name of the trail.)
10. Think of a sport that you like, such as football, soccer, baseball, or figure skating. What kinds of data might you store in a table? What would the rows and columns be? (Answers will vary. Usually each row will represent an athlete or team, and each column will be a statistic for that competitor.)

Written Work

In the lesson on signed numbers, we learned that signed numbers can be used to express increments and decrements. Use that knowledge to fill in two new columns in the table. The first two rows are done for you.

Date	Temperature	Rainfall	Change in temperature from previous day	Change in rainfall from previous day
June 1	82°	5 mm	—	—
June 2	72°	1 mm	-10°	-4 mm
June 3	73°	0 mm	1°	-1°
June 4	73°	0 mm		
June 5	77°	0 mm		
June 6	79°	4 mm		
June 7	81°	15 mm		
June 8	81°	13 mm		
June 9	82°	2 mm		
June 10	95°	1 mm		
June 11	91°	0 mm		
June 12	84°	0 mm		
June 13	73°	1 mm		
June 14	77°	5 mm		

Answers are on page 45 of the Solutions Guide.