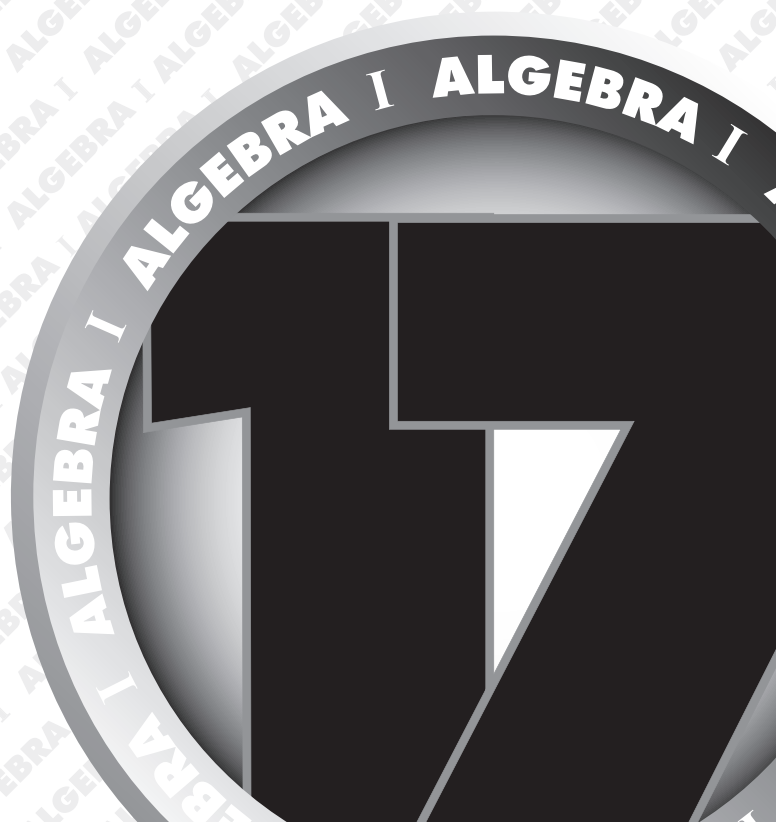


LEVEL SEVENTEEN

I
A
R
B
E
G
L
A

Developmental Mathematics

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1

COUNTING DOWN

- We know that as we count up, numbers increase **indefinitely** or **infinitely**

0, 1, 2, 3, 4, . . .

and as we count down, numbers decrease **definitely** or **finitely**

4, 3, 2, 1, 0

Counting down has stopped at 0.

We also know that:

- Numbers 0, 1, 2, 3, 4, . . . are called **whole** numbers.
 - Between any two consecutive whole numbers there is an indefinite number of numbers.
 - All numbers we know of, whole or otherwise, are called **rational** numbers.
- We still can count below zero:

1 below zero, 2 below zero, 3 below zero, . . .

and we can go **indefinitely**.

Also, between any two of these consecutive numbers there is an indefinite number of numbers. For example, between 0 and 1 below zero there is an endless number of numbers, each of which is less than 0 but not less than 1 below zero. Between 1 below zero and 2 below zero there is an endless number of numbers each of which is less than 1 below zero but not less than 2 below zero.

- In life, there are many situations where we need such numbers.

Some examples are:

- We consider 0° Fahrenheit as the starting point for temperature. We read temperature above 0° F and temperature below 0° F. It is clear that 25° below 0 is **less than** 14° below 0, while 19° below is **more than** 36° below 0.
- We consider sea level as the starting point of height. We consider sea level to be 0 feet, miles, or any other unit of length.

Then we count up or down.

Up: 15 ft. above sea level.
29 miles above sea level, ...etc.

Down: 29 ft. below sea level.
60 miles below sea level, ...etc.

APPLICATIONS

1. John said, "If you count up, there is no last number".

Do you agree? _____ Why?

2. Ted said, "If you count down, there is no last number".

Do you agree? _____ Why?

3. The average age in a class is 14 years old.

Sid is 16 years old and Sue is 13 years old.

How old are Sid and Sue compared to the average age in class?

4. A, B and C are three places.

A is 200 ft. below sea level.

B is 300 ft. higher than A.

C is 280 ft. lower than A.

How high are B and C compared to sea level?

5. a. How many numbers are there between 5 and 6? _____

b. How many decimals and fractions are there between 0 and 1? _____

c. How many numbers are there between 5 below zero and 6 below zero?

6. a. Which is higher, 160 feet below sea level or 295 feet below sea level?

b. Which is warmer, 6° below zero or 9° below zero?

c. Which is more, 8 below average or 19 below average?

2

SIGNED NUMBERS AND RELATED TERMS

- Now, we have two sets of numbers, numbers above 0 and numbers below 0.

Numbers which are more than 0 are called **positive** numbers.

Numbers which are less than 0 are called **negative** numbers.

The **+** sign has been assigned to **positive** numbers and the **-** sign to the **negative** numbers.

The sign is usually written to the left of the numeral. +5 -8

+5 is read positive 5.

-8 is read negative 8.

Thus, -17° F means 17 degrees Fahrenheit below 0.

and $+25^{\circ}$ F means 25 degrees Fahrenheit above 0.

- Sometimes positive and negative numbers are referred to as sign and/or signed numbers. The **+** sign or **-** sign associated with the numeral shows whether the number is above or below 0.

0, of course, is neither positive or negative.

- Signed numbers are usually used in situations which involve opposites. In business, you receive or pay money. If you use positive numbers for the amounts you receive, you use negative numbers for the amounts you pay. +5 means that you have received 5 units of money (cents, nickels, dimes, ...etc.) while -8 means that you have paid 8 units.

In sports, you win or lose games. If you use + for won and - for lost, +3 means that you have won 3 games, while -1 means that you have lost 1 game.

In medical research, signed numbers are used to report results. +25 means 25 successful cases, while -12 means 12 unsuccessful cases.

- Signed numbers, of course, may be whole numbers, fractions, or mixed numbers as shown in the following examples:

$$+\frac{5}{7}$$

$$-.95$$

$$+3\frac{4}{5}$$

$$-6.84$$

- Which is larger -3 or +2? • Which is smaller -5 or -3?
- 3 means 3 less than 0. -5 means 5 less than 0.
- +2 means 2 more than 0. -3 means 3 less than 0.

Thus, +2 is larger than -3.

Thus, -5 is less than -3.

APPLICATIONS

1. a. What does a positive number mean?

- b. What does a negative number mean?

2. a. Use symbols to write positive 6.

- b. Use symbols to write negative 4.

3. Give two life areas where positive and negative numbers may be used, and explain how such numbers may be used in each area.

- a. _____

- b. _____

4. a. Which is larger -5 or -8?

- b. Which is larger +4 or -6?

_____ Why?

_____ Why?

5. If you use + for "above average" and - for "below average", and Dan's grade is -7 while Sue's is +4.

- a. Who has more points?

- b. How many more?

6. A and B are two places.

A is +200 ft. high.

B is -100 ft. high.

How high is B compared to A?

7. We know that a. 32° Fahrenheit corresponds to 0° Centigrade.

and b. A change in 1° Centigrade is equivalent to a $\frac{9}{5}^{\circ}$ F change.

What temperature in Fahrenheit corresponds to -5° Centigrade?

- **Directed Numbers**

Sometimes signed numbers are called **directed numbers** because they tell the numbers and the direction from 0: above 0 or below 0.

- **Absolute Value**

A signed number is composed of a magnitude and a sign.

-6 is composed of the magnitude 6, and the - sign.

+6 is composed of the magnitude 6, and the + sign.

Both numbers have the same magnitude.

In a signed number, the magnitude is known as its **absolute value**.

Examples: $|-6| = 6$ $|+8| = 8$

- **Opposite Numbers**

Two signed numbers which have the **same magnitude**, but opposite signs, are called **opposite numbers**.

Examples: +8 and -8 +69 and -69 +.1 and -.1

You can simply state that every number, except 0, has an opposite.

- **Integers**

All **whole** numbers, their opposites and 0 put together are called **integers**.

- **Rational Numbers**

With our knowledge of numbers and their opposites, we go back to revise the meaning we have for the term **rational number**. The term is extended to mean any number (positive or negative) that may be expressed in fractional form. A fraction is a ratio of numbers. A rational number is a value that can be expressed as the ratio of whole numbers.

-4, $\frac{21}{3}$, $\frac{6}{7}$, $-1\frac{4}{9}$, -1.7 are different examples of **rational numbers**.

- Now let us review our mathematical vocabulary. If the meanings are not clear in your mind, review Level 11, Pages 3-6.

a) Numbers

b) Rational numbers

c) Counting numbers

d) Cardinal numbers

e) Ordinal Numbers

f) Whole Numbers

g) Integers

APPLICATIONS

1. a. What is the absolute value of +5? _____
b. Use symbols to write the equation. _____
c. Explain why your answer is correct.

2. a. What is the sum of the absolute values of +5 and -3? _____
b. What is the difference between the absolute values of +5 and -3? _____
3. a. What is the opposite of +8? _____
b. How are a number and its opposite related?

4. a. Give an example of an integer. _____
b. What type of numbers make up the set of integers?

5. Fred said, "Integers are rational numbers".
Do you agree? _____ Why? _____

6. Ann said, "Rational numbers are integers".
Do you agree? _____ Why? _____

7. a. What different names can you give to the number +6?

b. What different names can you give to the number -6?

3

COUNTING UP, COUNTING DOWN

- If you count up, numbers **increase**. Each number is 1 more than the previous number.

Example: +5, +6, +7, +8, +9 ...
 0, +1, +2, +3, +4 ...
 -2, -1, 0, +1, +2 ...
 -8, -7, -6, -5, -4 ...

If you count down, numbers **decrease**. Each number is 1 less than the previous number.

Example: +5, +4, +3, +2, +1 ...
 +3, +2, +1, 0, -1 ...
 0 -1, -2, -3, -4 ...
 -4, -5, -6, -7, -8 ...

- Start with +5 and count 2 up.

+5, +6, +7

Start with 0 and count 4 up.

0, +1, +2, +3, +4

Start with -3 and count 5 up.

-3, -2, -1, 0, +1, +2

Start with -2 and count 5 up.

-2, -1, 0, +1, +2, +3

- Start with +4 and count 3 down.

+4, +3, +2, +1

Start with 0 and count 5 down.

0, -1, -2, -3, -4, -5

Start with -2 and count 3 down.

-2, -3, -4, -5

Start with +1 and count 4 down.

+1, 0, -1, -2, -3

APPLICATIONS

1. Start with +5 and count 3 up.

2. Start with 0 and count 4 up.

3. Start with -4 and count 4 up.

4. Start with -6 and count 6 up.

5. Start with -1 and count 3 up.

6. Start with -5 and count 4 up.

7. Start with -8 and count 6 up.

8. Start with +5 and count 3 down.

9. Start with 0 and count 4 down.

10. Start with -6 and count 2 down.

11. Start with +4 and count 4 down.

12. Start with +8 and count 3 down.

13. Start with +6 and count 8 down.

14. Start with -5 and count 9 up.

15. Start with -5 and count 9 down.

4

WHAT IS THE LAST NUMBER?

- You started with +4 and counted 3 up.
Is the last number positive or negative? _____
What is the last number? _____
- You started with +4 and counted 4 down.
Is the last number positive or negative? _____
What is the last number? _____
- You started with +4 and counted 6 down.
Is the last number positive or negative? _____
What is the last number? _____
- You started with -4 and counted 8 down.
Is the last number positive or negative? _____
What is the last number? _____
- You started with -8 and counted 8 up.
Is the last number positive or negative? _____
What is the last number? _____
- You started with +6 and counted 18 down.
Is the last number positive or negative? _____
What is the last number? _____
- You started with +300 and counted 200 down.
Is the last number positive or negative? _____
What is the last number? _____
- You started with -28 and counted 18 down.
Is the last number positive or negative? _____
What is the last number? _____
- You started with -192 and counted 78 up.
Is the last number positive or negative? _____
What is the last number? _____

In all the examples, you start by finding the sign of the last number.
To find its magnitude, you add or subtract as the situation requires.

APPLICATIONS

Find the last number:

- | | | | |
|-----------------------|-------|-------------------------|-------|
| 1. +4 and 7 up | _____ | 2. +4 and 6 up | _____ |
| 3. +8 and 13 up | _____ | 4. +9 and 25 up | _____ |
| 5. -6 and 3 up | _____ | 6. -12 and 6 up | _____ |
| 7. -8 and 8 up | _____ | 8. -6 and 6 up | _____ |
| 9. -4 and 6 up | _____ | 10. -5 and 4 up | _____ |
| 11. -25 and 35 up | _____ | 12. -13 and 35 up | _____ |
| 13. -23 and 16 up | _____ | 14. -28 and 19 up | _____ |
| 15. +8 and 4 down | _____ | 16. +9 and 9 down | _____ |
| 17. +8 and 8 down | _____ | 18. +6 and 8 down | _____ |
| 19. +2 and 31 down | _____ | 20. 0 and 14 down | _____ |
| 21. +1 and 27 down | _____ | 22. -1 and 3 down | _____ |
| 23. -8 and 7 down | _____ | 24. -9 and 25 down | _____ |
| 25. -26 and 14 down | _____ | 26. -27 and 35 down | _____ |
| 27. +15 and 28 down | _____ | 28. -17 and 14 up | _____ |
| 29. -38 and 26 down | _____ | 30. -20 and 15 up | _____ |
| 31. -1 and 200 down | _____ | 32. -1 and 200 up | _____ |
| 33. -3 and 26 up | _____ | 34. -15 and 15 up | _____ |
| 35. +281 and 281 down | _____ | 36. -300 and 28 up | _____ |
| 37. +60 and 360 up | _____ | 38. -219 and 19 down | _____ |
| 39. -18 and 218 down | _____ | 40. +236 and 36 up | _____ |
| 41. +31 and 4 up | _____ | 42. -75 and 75 down | _____ |
| 43. 0 and 25 down | _____ | 44. -275 and 574 up | _____ |
| 45. -95 and 95 down | _____ | 46. -961 and 275 up | _____ |
| 47. +434 and 759 down | _____ | 48. -1325 and 2455 up | _____ |
| 49. -3973 and 4521 up | _____ | 50. +7539 and 8704 down | _____ |