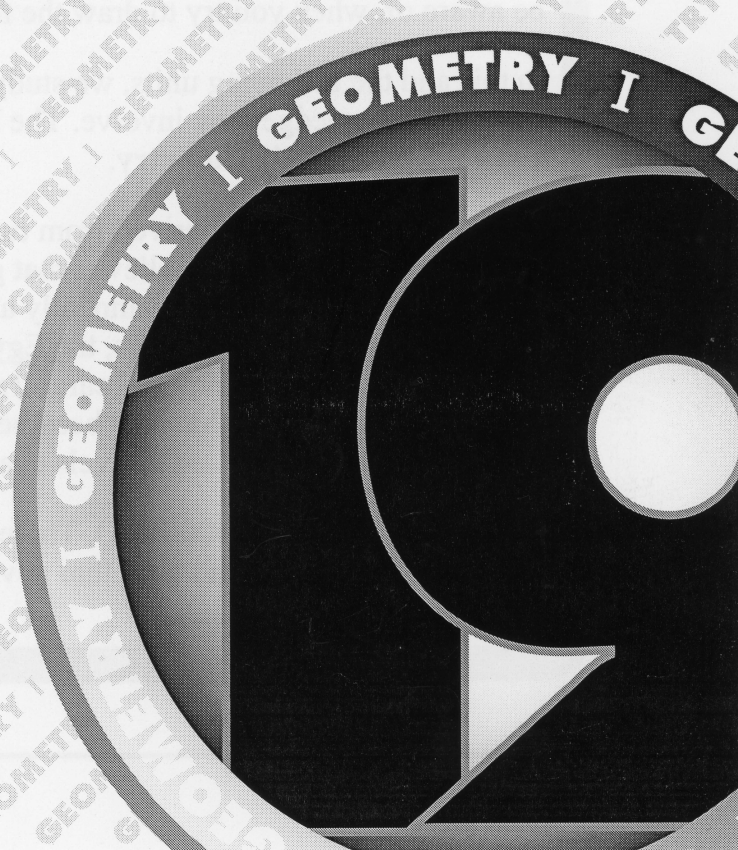


LEVEL

GEOMETRY

Developmental Mathematics

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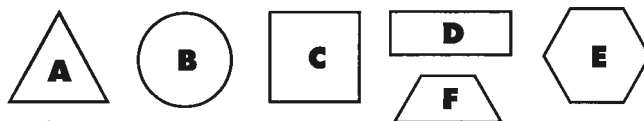


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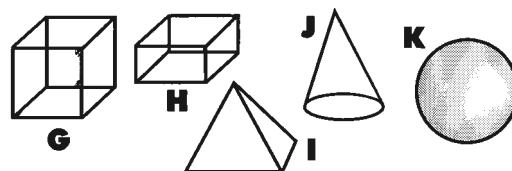
WHAT IS GEOMETRY?

You have already become familiar with certain shapes.



Some of these shapes are two-dimensional (flat) such as those shown: A-F.

Other shapes are three-dimensional (solid) such as those shown: G-K.



You can easily recognize a triangle, a circle, a cube, a square, ...etc. Suppose you are asked “What shape is Figure B?”

Is it a circle? If you say, “Yes”, the next question will be “Why do you say so?” If you say “No”, you still will be asked “Why do you say so?”

You cannot answer the question, “Why do you say so?” unless you know what a circle is.

Do you know what a circle is?

A circle is the shape traced if a constant radius is kept about a given center point in a plane.

Carefully study the given definition to determine if you can use it all the time.

Each of the figures you see above (and many others) has a certain definition you have to be aware of, when you try to draw the figure.

In this and the following units, we study such figures systematically; what they are, and what knowledge they involve. The branch of Mathematics that deals with such knowledge is called **Geometry**.

The word **Geometry** is derived from two Greek words: **geo** meaning earth, and **meterin** meaning to measure. It is believed that geometry originated in Egypt about 1350 B.C. The Nile River used to flood annually and remove any boundary markers. To reestablish these boundaries, the Egyptians developed mathematical methods based on systematic observations and careful measurements. Through trial and error, the methods formulated in this way became very useful tools. With them, the Egyptians built The Pyramids that are marvels even by modern standards.

Later in history (about 600 B.C.), Thales, Pythagoras, Plato and Aristotle, helped in developing principles and laws that govern knowledge in the field of Geometry.

One of the greatest of all scholars was Euclid, in Egypt (300 B.C.), who was a teacher of Mathematics at the University of Alexandria. He collected all geometric knowledge of his day and organized it into a logically sound body of principles. His great work is known as *The Elements*.

In this level, we will study Geometry as organized by these scholars. We will also study the practical applications of the knowledge we acquire.

Our study of Geometry will be covered in two levels of the individualized *Developmental Mathematics* curriculum.

Level 19: Geometry I

In this level, we will study principles and knowledge basic to the study of Geometry.

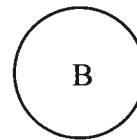
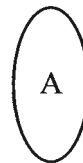
Level 20: Geometry II

In this level, we will study the figures which can be constructed (drawn) on a flat surface.

APPLICATIONS

1. a. Which figure is a circle? _____

b. Why do you say so? _____



2. a. Draw a square?

b. Explain how you drew it. _____

3. a. What does "defining a term" mean? _____

b. Why is it important to define the terms we use in geometry?

4. Who uses geometric knowledge in his/her occupation or profession?

Name as many as you can.

2

POINT - LINE - PLANE**POINT**

Have you ever seen a point? What you see to the right is not a point. It is a dot which is a deposit of ink. A geometric point has no size. It only has position. The dot we make with pen or pencil is not the point itself, but merely a physical representation of the point.

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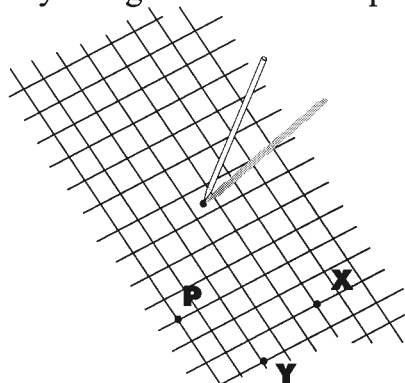
If we try to define a point, we will find it very difficult. We only have a notion of what a point is rather than a clear definition of the point.

This is why the term **point** is considered an undefined term.

Good examples of geometric points are the corners of this sheet of paper. You can locate them. The bright spots we see in the sky at night are also examples.

A navigation officer on a plane or a ship may put a dot on his chart to represent the exact location in space of his craft.

It has been agreed upon to label the dot that represents a point with a capital letter as shown to the right.

**LINE**

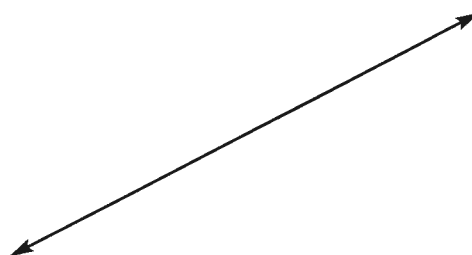
Another term which is hard to describe precisely is the **line**. I am sure that you see the line where the ceiling and the wall meet. The edge of this sheet of paper is also another example.

The line has only one dimension, that of length, and like a point, has no width or height.

.....

The set of points represented by the dots you see to the right give the notion of a line. You can, of course, think of more points on the same line. In fact, you can think of an unlimited number of points. Between any two points there is a point, simply because one point does not have length, or any other dimension.

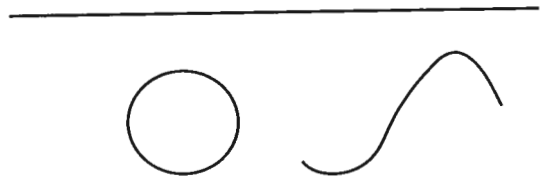
Lines have one dimension, that is of length, but they do not have specific measure of length. A line is considered to extend indefinitely.



There are two types of lines:

Those which may be represented by a taut string and those which may not.

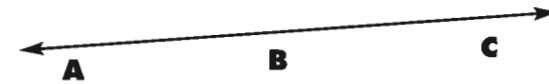
The first type is a straight line, while the other is a curve. The curve may be closed or not.



However, it has been agreed upon that unless otherwise stated, the term **line** refers to a straight line. Also a line may be named by naming any two different points of the line. The line represented to the right may be identified by all of the following:

$\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{CA}$, $\overleftrightarrow{CB}$

A straight line extends indefinitely in both directions. Any two points identify a line that extends in both directions beyond the two points. Draw the line $\overleftrightarrow{MN}$.



LINE SEGMENT

A **line segment** is any set of points on a line between any two points of the line. The two points are the two endpoints, or simply the ends and are considered part of the line segment.



The length of a line segment is the distance from one endpoint to the other. The length of a line segment may be represented by a number which tells how many units of length are contained in it.

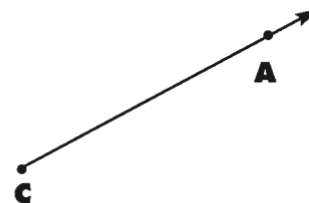
A line segment is named by its two endpoints. The line segment represented above is named $\overline{AB}$ or $\overline{BA}$.

RAY

A part of a line with one endpoint only is called a **ray**.

It identifies a certain direction.

A ray may be named by its endpoint and another point on the ray.



The ray represented to the right may be named $\overrightarrow{CA}$, but not $\overrightarrow{AC}$.

PLANE

We describe the top of a table or the floor of a room as **flat** surfaces. Other examples are the surface of a sheet of plywood, a chalkboard or a wall. These examples give the notion of a **plane**. Like the line, the plane extends indefinitely. However, the plane extends in all directions.

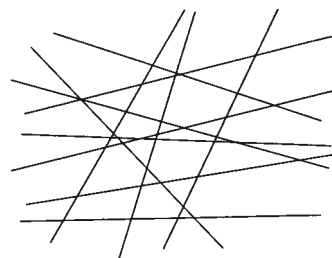
A plane contains an unlimited number of points.

A plane represented by the surface of this sheet of paper contains an infinite number of points, some of which are represented to the right.



A plane also contains an unlimited number of lines.

Any two points identify one line. There are infinite numbers of points in a plane. Thus, there is an infinite number of lines in a plane.



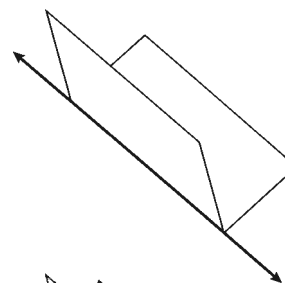
The two points represented to the right are of course, in the plane represented by the surface of this paper.



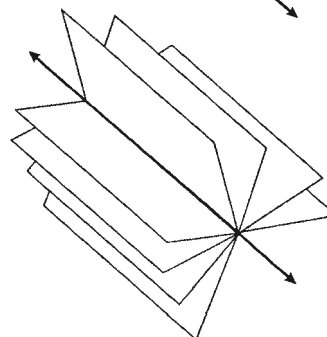
Draw the line that the two points identify. This line also lies in the same plane.

In general, if a line contains two points of a plane, it lies in the plane.

Take a piece of paper and fold it and then open it as shown to the right. You have two planes each of which passes through the same line.



You can, of course, have as many planes as you want that pass through the same line as shown to the right.



In general, through any given line, an infinite and unlimited number of planes pass.

APPLICATIONS

1. What notions do you have of a geometric point?

2. What notions do you have of a geometric line?

3. A straight line has the dimension of length but not a measure of length. Explain.

4. Complete the following statements:

- a. Any two points identify a _____.
- b. A line extends _____.

5. What is a line segment?

6. a. A certain line segment is 5 inches long.

What does this mean? _____

b. Draw a line segment $3\frac{1}{2}$ inches long.

7. What is a ray? _____

8. a. How many rays may have the same endpoint? _____.

b. Two rays headed in opposite directions from the same endpoint identify _____.

9. What notions do you have of a geometric plane?

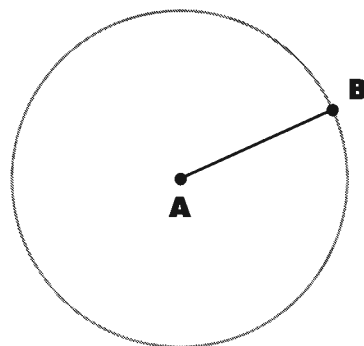
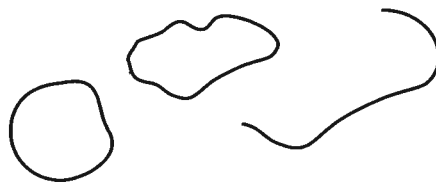
10. a. How many lines may be in a plane? _____

b. How many planes may pass through the same line? _____

3

CIRCLE - RADIUS

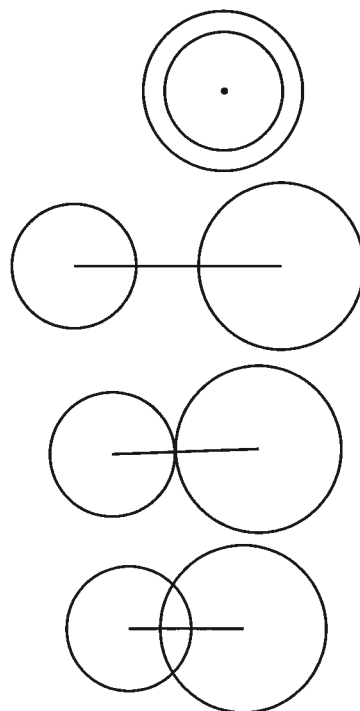
- If you move a pencil on paper with continuous change in direction, you trace a curve. To the right are examples of curves. They are **plane curves**. Each curve lies completely in the same plane identified by the surface of this paper.
- Let us show a certain type of curve. A is a point. $\overline{AB}$ is a line segment. With your pencil, start at B and move, keeping **the distance from A the same all the time**. You eventually return to B. You have traced a closed curve that is a **circle**.
- A is the **center** of the circle and $\overline{AB}$ is a **radius**.
- Let us describe the circle precisely. Let us **define** the circle.
- A circle is a closed curve in a plane, each point of which is the same distance from a fixed point in the plane.
- The radius of a circle is the distance from the fixed point in the center to any point on the circle.
- To make it easy to construct (draw) circles or portions of circles, we use a compass.
- Two circles may have the same center. Such circles are called **concentric circles**.



Two circles may have different centers and have no points which are common to both, as shown to the right. This happens when the distance between the centers is longer than the sum of the two radii.

Two circles may touch each other, as shown to the right. This happens when the distance between the two centers is **exactly equal** to the sum of the two radii.

Two circles may intersect as shown to the right. This happens when the distance between the two centers is less than the sum of the two radii. The two circles intersect at two points.



APPLICATIONS

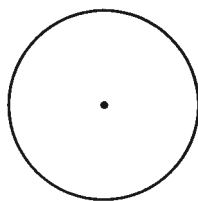
1. Draw a circle whose center is A and whose radius is OA.



2. Draw a circle whose center is A and whose radius is a half inch.



3. Find the radius of the circle given below.



4. Draw any two concentric circles.

5. Draw two circles whose centers are P and Q and that touch each other.



6. Draw two intersecting circles whose centers are O and P.



7. Draw a circle whose center is M and whose radius is $\frac{3}{4}$ ".
Draw another circle whose center is N and whose radius is 1".

Do the two circles intersect? _____

Why?



4

CHORD - ARC

- Take a Point A on a circle with center O. Join OA and extend until it meets the circle at B. AB is called a **diameter**. Its length is twice that of the radius of the circle. Any diameter of a circle divides it into two **semicircles**.

- A part of the circle is called an **arc**. Any two points on a circle divide the circle into two arcs. One arc may be smaller than the other. The two arcs may also be equal.

- Take any two points, A and B on a circle. Join $\overline{AB}$. The line segment $\overline{AB}$ is called a **chord**.

If a chord passes through the center it is called a **diameter**.

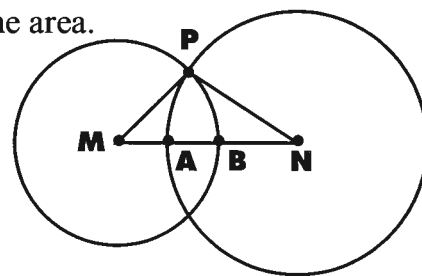
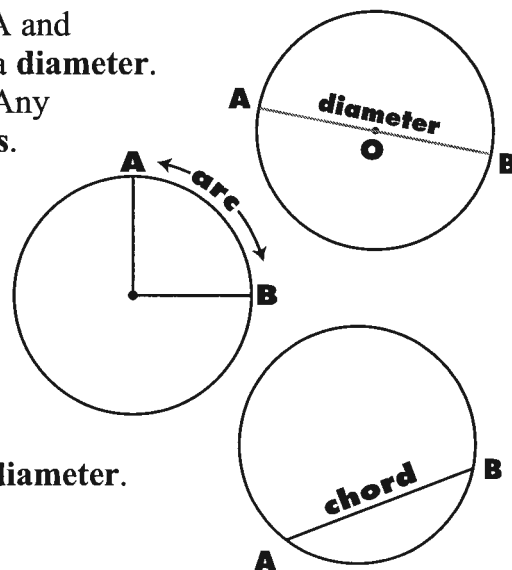
- A and B are two points.

Locate a point which is 1" from A and $\frac{3}{4}$ " from B.

- open your compass to 1".
- with the tip of the compass on A draw an arc.
- open your compass to $\frac{3}{4}$ ".
- with the tip of your compass at B draw an arc.
- the two arcs intersect at two points. Name them M and N.

Either M or N is the required point to complete the area.

- In the figure to the right, M and N are centers of the two circles. $\overline{MN}$ intersects the circles at A and B.



$$\overline{AB} = \overline{PN} - \overline{BN} \quad \text{Why?}$$

$$\overline{AB} = \overline{PM} - \overline{AM} \quad \text{Why?}$$

APPLICATIONS

1. Locate a Point which is 2 centimeters from A and 3 centimeters from B.



2. Locate a Point O above $\overline{MN}$ which is the same distance from M and N.



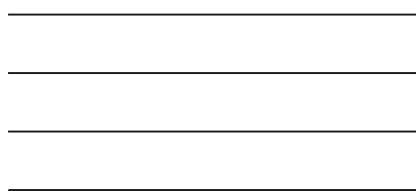
Can you find another point? _____

How many points can you locate?

3. Locate a Point R above $\overline{PQ}$ such that $\overline{RP} = \overline{PQ} = \overline{QR}$.



4. In the figure below, A and B are the centers. A, M, N and B are co-linear. (Points in the same straight line)
Show that: $\overline{AP} - \overline{AM} = \overline{BP} - \overline{BN}$



5. The figure to the right is just a sketch. In this figure, one circle passes through the center of the other. The line of Centers O and P intersects the smaller circle at Q. If the radii of the two circles are 5" and 7", find the length of $\overline{PQ}$.

Show why. _____



6. The figure to the right is just a sketch. Two circular arcs of Centers O and P intersect at A. $\overline{OP}$ intersects the circles at M and N.

If $\overline{PA} = 6"$, $\overline{OA} = 9"$, $\overline{MN} = 2"$, find the of length $\overline{OP}$. Show why.

